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The equilibrium of rubble-pile satellites: The Darwin and Roche ellipsoids for gravitationally held granular aggregates

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ABSTRACT

Many new small moons of the giant planets have been discovered recently. In parallel, satellites of several asteroids, e.g., Ida, have been found. Strikingly, a majority of these new-found planetary moons are estimated to have very low densities, which, along with their hypothesized accretionary origins, suggests a rubble internal structure. This, coupled to the fact that many asteroids are also thought to be particle aggregates held together principally by self-gravity, motivates the present investigation into the possible ellipsoidal shapes that a rubble-pile satellite may achieve as it orbits an aspherical primary. Conversely, knowledge of the shape will constrain the granular aggregate's orbit—the closer it gets to a primary, both primary's tidal effect and the satellite's spin are greater. We will assume that the primary body is sufficiently massive so as not to be influenced by the satellite. However, we will incorporate the primary's possible ellipsoidal shape, e.g., flattening at its poles in the case of a planet, and the proloidal shape of asteroids. In this, the present investigation is an extension of the first classical Darwin problem to granular aggregates. General equations defining an ellipsoidal rubble pile's equilibrium about an ellipsoidal primary are developed. They are then utilized to scrutinize the possible granular nature of small inner moons of the giant planets. It is found that most satellites satisfy constraints necessary to exist as equilibrated granular aggregates. Objects like Naiad, Metis and Adrastea appear to violate these limits, but in doing so, provide clues to their internal density and/or structure. We also recover the Roche limit for a granular satellite of a spherical primary, and employ it to study the martian satellites, Phobos and Deimos, as well as to make contact with earlier work of Davidsson [Davidsson, B., 2001. Icarus 149, 375–383]. The satellite's interior will be modeled as a rigid-plastic, cohesion-less material with a Drucker–Prager yield criterion. This rheology is a reasonable first model for rubble piles. We will employ an approximate volume-averaging procedure that is based on the classical method of moments, and is an extension of the virial method [Chandrasekhar, S., 1969. Ellipsoidal Figures of Equilibrium. Yale Univ. Press, New Haven] to granular solid bodies.

1. Introduction

Roche (1847) first considered the problem of finding the equilibrium shape of a fluid satellite of a spherical planet, identifying ellipsoidal equilibrium shapes now known as the Roche ellipsoids. Later, Darwin (1906) introduced a non-trivial generalization of the Roche problem: the characterization of equilibrium shapes of two tidally interacting fluid bodies that rotate about each other on circular orbits. Within the context of his calculation, Chandrasekhar (1969) showed that, at least in the case of fluids, there are only two possible scenarios in which such ellipsoidal equilibrium shapes may be found. The first is when the aspherical primary is massive enough to warrant neglecting the tidal effects due to the satellite. In the other case, both objects are congruent, i.e., with the same shape, mass and in a symmetric orientation. Indeed, Darwin (1906) himself had emphasized this dichotomy. We will refer to this natural classification as the *first* and *second* Darwin problems, whose solution, for an *inviscid fluid*, yields the Darwin sequence of ellipsoids (Chandrasekhar, 1969).

Recently, several small inner moons of the giant planets have been discovered. Their estimated low densities, often lower than or comparable to water's, suggests that these objects may be either granular aggregates, or highly porous cellular "honeycomb"-like structures. While the former, in the absence of cohesion, has no tensile strength and is held together only by its own gravity, the latter is able to withstand a certain amount of tension. It is, however, believed that these newly uncovered satellites may have formed via an accretionary process either from ring particles, as in the case of Saturn's moons (Porco et al., 2007), or from the debris left over from some past catastrophic event, e.g., Neptune's capture of Triton (Banfield and Murray, 1992). There is thus a need to generalize the first of the two classical Darwin problems introduced above to *rubble-pile* satellites of *oblate* planets. In fact, the finding

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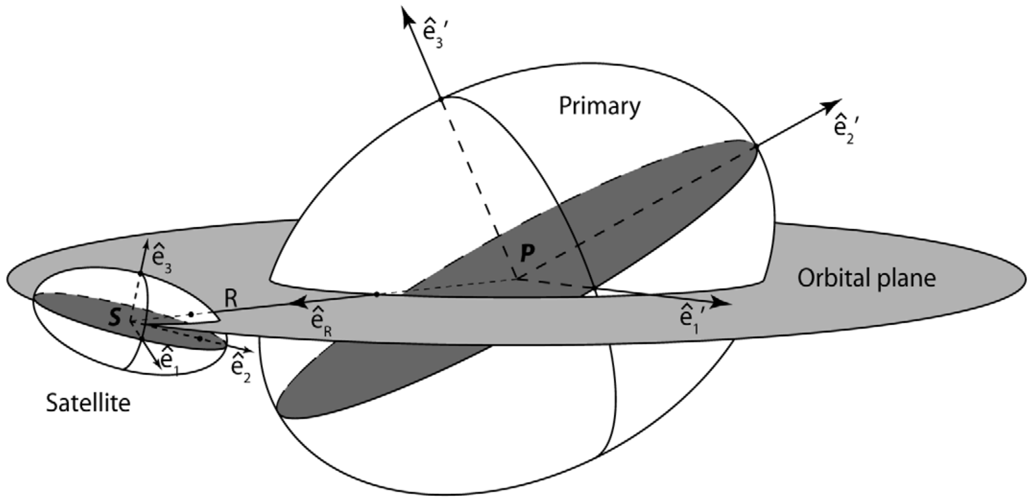

Fig. 1. The general configuration of an ellipsoidal satellite of an ellipsoidal primary. The unit vector $\hat{e}_R$ locates the satellite with respect to the primary's center.

of asteroidal satellites, many of which are suspected particle aggregates, strongly suggests the need to consider also elongated triaxial primaries. The general scenario is displayed in Fig. 1.

Rubble piles, while much weaker than coherent structures, are able to sustain shear stresses due to internal friction. This allows a range of equilibrium satellite shapes to be possible at a given planetary distance. Conversely, for a given shape, the satellite's orbits on which it may persist in equilibrium are not necessarily unique. In the sequel, we will obtain general equations that describe the equilibrium landscape of a triaxial-ellipsoidal, tidally-locked, rubble-pile satellite on a circular orbit around a triaxial-ellipsoidal primary. The formulation will then be specialized to investigate the moons of the giant planets. In Section 5, the Roche limit for a granular aggregate, i.e., the critical distance at which a rubble-pile satellite may orbit a spherical planet, will also be obtained as a special case and will then be employed to study the two satellites of Mars. In this context, Holsapple and Michel (2006, 2008) have recently considered the Roche limit for solid bodies, employing the static version of Signorini's theory of stress means (Truesdell and Toupin, 1960, p. 574). We compare briefly with their results in Section 5.2.

We will employ a volume-averaging procedure that is really a generalization of Chandrasekhar's (1969) virial method to the statics, and dynamics, of solid objects. Previously, this volume-averaging procedure has been employed to investigate tidal disruption during planetary flybys (Sharma, 2004; Sharma et al., 2006), equilibrium shapes and dynamical passage into them for asteroids in pure spin (Sharma et al., 2005a, 2005b, 2008), and the Roche limit for rubble piles (Sharma et al., 2005b; Burns et al., 2007). A good match with available computational and analytical results was achieved. In fact, in the case of rubble piles in pure spin, the equilibrium landscape obtained from volume-averaging matched perfectly Holsapple's (2001) exact results that were based on rigorous limit analysis (Chen and Han, 1988) often employed in rigid-plasticity.

We develop the governing equations next.

2. Volume-averaging

In this section, we present the main equations obtained from an application of the volume-averaging procedure. More details about these derivations may be found in Chandrasekhar (1969), Sharma et al. (2006), or Sharma et al. (2008).

In case of a tidally interacting satellite, there are principal-axes coordinate systems associated with both the satellite and the primary. No particular system is better suited to evaluate all quantities that will appear below. In addition, many different relative orientations of the primary and the satellite are possible. Thus, it is advantageous to follow a coordinate-independent tensor-based approach. We will develop general equations, applicable to all primary-satellite configurations, as far as possible, and only specialize to a particular primary-satellite configuration at the very end. We offer a very short primer on tensors and operations with them in Appendix A. More information may be obtained from Knowles (1998). The reader should also refer to Appendix A for notations followed in this paper.

2.1. Governing equations

We consider a finite-sized satellite in motion about a primary also of finite spatial extent. Much work has been done on the dynamical problem of relative equilibria in the two finite rigid-body problem, and we refer the interested reader to Kinoshita (1972), and more recently to Scheeres (2006). Indeed, the two finite rigid-body problem will be relevant when extending the present work to the equilibrium shapes in a binary system consisting of comparable masses, i.e., the second Darwin problem introduced in Section 1. In this paper, we focus on the first Darwin problem, wherein the primary is much more massive than the satellite. Thus, we neglect the motion of the primary's center of mass. We also make the further assumption that the satellite moves around the primary at a much faster rate than the primary around the Sun.

The stress $\boldsymbol{\sigma}$ inside a deformable body, such as a satellite, may be obtained by solving the Navier equation (see, e.g., Fung, 1965)

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathsf{b} = \rho(\ddot{\mathsf{x}} + \ddot{\mathsf{R}}), \tag{1}$$

where ρ is the density, b the body force, x the location of a material point with respect to the satellite's center of mass, and R locates the satellite's moving mass center with respect to the primary's fixed center of mass (see Fig. 1). To solve the above partial differential equation, one must provide appropriate boundary conditions, and compatibility equations that incorporate the material's constitutive behavior. Note that, within the approximations introduced in the preceding paragraph, $\ddot{\mathsf{x}} + \ddot{\mathsf{R}}$ is the total acceleration of a material point and $\ddot{\mathsf{x}}$ is the acceleration relative to the satellite's mass center. The body force includes in our case the tidal effects of the primary on the satellite, and of the satellite's internal gravity on itself. Except in the simplest of geometries, loading conditions, rheologies and small deformations, solving for the exact stresses is often analytically intractable, and even computationally very involved. Considering the intricate constitutive nature of planetary