Icarus 197 (2008) 211–220

Contents lists available at ScienceDirect

Icarus

www.elsevier.com/locate/icarus

Growth and evolution of small porous icy bodies with an adaptive-grid thermal evolution code
I. Application to Kuiper belt objects and Enceladus

Dina Prialnik*, Rainer Merk

Department of Geophysics and Planetary Sciences, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel

ARTICLE INFO

Article history:
Received 29 November 2007
Revised 19 March 2008
Available online 16 April 2008

Keywords:
Comets
Comets, nucleus
Enceladus
Kuiper belt
Thermal histories

ABSTRACT

We present a new 1-dimensional thermal evolution code suited for small icy bodies of the Solar System, based on modern adaptive grid numerical techniques, and suited for multiphase flow through a porous medium. The code is used for evolutionary calculations spanning 4.6×10^9 yr of a growing body made of ice and rock, starting with a 10 km radius seed and ending with an object 250 km in radius. Initial conditions are chosen to match two different classes of objects: a Kuiper belt object, and Saturn's moon Enceladus. Heating by the decay of ^{26}Al, as well as long-lived radionuclides is taken into account. Several values of the thermal conductivity and accretion laws are tested. We find that in all cases the melting point of ice is reached in a central core. Evaporation and flow of water and vapor gradually remove the water from the core and the final (present) structure is differentiated, with a rocky, highly porous core of 80 km radius (and up to 160 km for very low conductivities). Outside the core, due to refreezing of water and vapor, a compact, ice-rich layer forms, a few tens of km thick (except in the case of very high conductivity). If the ice is initially amorphous, as expected in the Kuiper belt, the amorphous ice is preserved in an outer layer about 20 km thick. We conclude by suggesting various ways in which the code may be extended.

1. Introduction

Whether or not liquid water could have been present in small—presently icy—bodies of the Solar System during their early stages of evolution, and if so, under which conditions, is still debated. The question whether and to what extent are these bodies (comets, Kuiper belt objects and icy satellites of the distant planets) pristine bodies that hold clues to the formation of the Solar System is also still open. And finally, would it be possible for such bodies to have had liquid cores, but at the same time to have preserved their outer layers in pristine form? These are some of the questions that have prompted the development of increasingly elaborate thermal evolution models, which are steadily growing in number. The effort is twofold: understanding and providing a mathematical formulation of these processes on the one hand, and incorporating them in numerical simulations of structure and evolution, on the other.

Models show that incorporating radiogenic heating by short-lived radionuclides, such as ^{26}Al, provides sufficient energy for melting the ice and even increasing the water temperature to boiling point (e.g., Merk and Prialnik, 2006). These works, spanning a period of over 25 years, have been recently reviewed by Podolak and Prialnik (2006), Jewitt et al. (2007) and McKinnon et al. (2007). The drawback of these studies is that they do not consider the three phases of H_2O in a complete and consistent manner. In particular, water is assumed to be produced when the ice temperature exceeds 273 K, but its possible evaporation or flow through the porous medium is ignored. It is thus unclear whether water accumulates in the hot central part of the body, or evaporates, or else flows into colder regions and refreezes. The purpose of the present study is to follow the evolution of the liquid water and its possible migration through the pores in order to derive the structure of the body and its variation with depth.

Mathematical modeling of the evolution of any small body made of ice and dust (rock), involves a system of coupled nonlinear partial differential equations (PDE) and associated initial and boundary values. In general, the system does not have a closed analytic solution, and thus one has to resort to numerical integration techniques. The differential equations are replaced by finite-difference equations on a grid of points that cover the range of integration. Time is divided into time steps. Since the equations have to satisfy boundary conditions at two different points of the spatial grid, we are faced with a two-point boundary value problem. The numerical solution requires relaxation techniques that demand several iterations. An iteration consists of adjusting the values of a desired function on all the grid points simultaneously,

* Corresponding author.
E-mail address: dina@planet.tau.ac.il (D. Prialnik).

0019-1035/$ – see front matter
doi:10.1016/j.icarus.2008.03.024

until they successively satisfy the finite-difference equations and the prescribed boundary conditions to within a specified accuracy. The procedure is repeated at each time step. The relaxation method works best when the solution is smooth both in time and in space. But, while time steps may be adjusted as required by the evolutionary processes involved, the spatial grid is predetermined. This gives rise to severe computational problems.

First, if the body under consideration is growing or shrinking, the range of integration is obviously not fixed, but must be continually changed. The rough technique, often used, of adding or removing grid points introduces significant numerical noise and requires additional ad-hoc assumptions. Moreover, steep gradients—of pressure and temperature—arise close to the surface, where the solar energy is absorbed. The surface layer, however, is soon eroded by sublimation of ice and drag of the dust embedded in it. Since near the surface grid intervals are normally many orders of magnitude smaller than in the deep interior, this means that a thick outer layer must be divided at the outset into very small intervals. Not only will this affect the accuracy of the numerical solution, but the resulting large number of grid points will render the computation time prohibitively large (since matrix inversion is required at each iteration of each time step). This difficulty may be circumvented by implementing a moving boundary technique.

Secondly, phase transition fronts, which are usually very thin, i.e., involve sharp gradients, move through the grid as the body evolves. Thus a suitable grid at a given time, will cease to be so at a later stage. The allocation of grid points must therefore be done dynamically. To ensure a smooth solution of the equations, it is not sufficient to adjust zoning occasionally, after the system has converged, but rather during the relaxation process itself. Such adaptive zoning may be accomplished by automatic allocation of grid points. Essentially, it requires an additional equation (ordinary or differential) that formulates mathematically the constraints imposed on the grid. For example, spacing must be such that the temperature difference between two neighboring points does not exceed a fixed fraction of the mean.

Adaptive grid techniques have been developed precisely for this type of evolutionary problems, but have not yet been implemented in modeling small Solar System bodies. In the computation of the evolving structure of stars, such methods have been introduced already a few decades ago (Eggleton, 1971). As in comets (icy bodies), they are meant to overcome the difficulties of moving reaction fronts, nuclear reaction fronts in stars being the analogues of sublimation or crystallization fronts in icy bodies. Although the physics is radically different, these are essentially identical numerical problems. It is therefore to be expected that numerical methods that have been successfully applied to modeling the evolution of stars will be equally successful when applied to the evolution of comet nuclei, Kuiper belt objects and icy satellites.

In Section 2 we describe in some detail the thermal evolution code, the physics involved and the numerical procedure. In Section 3 we describe the results of long-term evolutionary calculations for two different prototypes: a Kuiper belt object and an object with the physical characteristics of Enceladus in Saturn's orbit around the Sun. Our focus will be on the internal structure of these objects. A brief discussion and our conclusions follow in Section 4.

2. The thermal evolution code

2.1. Assumptions and definitions

Consider a spherical body of mass M and radius R, composed of consolidated water ice and dust grains that form a porous matrix. Consider further *four* different phases of H_2O—amorphous ice, crystalline ice, liquid and vapor—and transitions between them. We thus have 5 different substances (components) that we denote by subscripts: d—dust; a—amorphous water ice; c—crystalline water ice; ℓ—liquid water; v—water vapor.

The porous medium properties are defined by the following parameters:

- *Density* ρ = mass per unit volume of the body (known also as *bulk density*) is defined (locally) for each component, as well as for the total:

$$\rho = \rho_d + \rho_a + \rho_c + \rho_\ell + \rho_v. \tag{1}$$

- *Specific density* ϱ = mass per unit volume of solid material of a particular species is taken from the literature for each of the solid components, as well as for the liquid water, which is assumed incompressible: ϱ_d, ϱ_a, ϱ_c and ϱ_ℓ.
- *Porosity* ψ (dimensionless) = void volume per unit volume of the body:

$$\psi = 1 - \left(\frac{\rho_d}{\varrho_d} + \frac{\rho_a}{\varrho_a} + \frac{\rho_c}{\varrho_c}\right). \tag{2}$$

- *Saturation* χ (dimensionless) = volume occupied by the liquid per void volume:

$$\chi = \frac{\rho_\ell/\varrho_\ell}{\psi}. \tag{3}$$

- *Pore size* r_p (length) = average radius of a pore, which may be supplied or derived from a given pore size distribution.
- *Surface to volume ratio* S (inverse of length) = total surface area of pores per given bulk volume, may be calculated when a model of the porous structure is supplied. Generally, $S \propto 1/r_p$.

We note that, given the properties of the different phases of H_2O and the transitions between them, it is possible to have coexistence of amorphous and crystalline ice and of crystalline ice, liquid water and vapor, but not simultaneous occurrence of all four phases. It will prove useful to define a density ρ_w as

$$\rho_w = \rho_c + \rho_\ell \tag{4}$$

as the density of the "non-gaseous" phase of crystallized H_2O. The liquid fraction of ρ_w will be taken as a smoothed step function of the temperature around the melting temperature T_m:

$$\rho_\ell = \frac{\rho_w}{1 + e^{\beta(1-T/T_m)}}, \tag{5}$$

where $\beta \gg 1$ determines the steepness of the smoothed step function.

2.2. Conservation equations

2.2.1. Mass

$$\frac{\partial \rho_a}{\partial t} = -\lambda(T)\rho_a, \tag{6}$$

$$\frac{\partial \rho_w}{\partial t} + \nabla \cdot \mathbf{J}_\ell = \lambda(T)\rho_a - q_v, \tag{7}$$

$$\frac{\partial \rho_v}{\partial t} + \nabla \cdot \mathbf{J}_v = q_v, \tag{8}$$

$$\frac{\partial \rho_d}{\partial t} = 0 \quad \text{for the time being.} \tag{9}$$

Here $\lambda(T) = 1.05 \times 10^{13} e^{-5370/T}$ s^{-1} is the rate of crystallization of amorphous ice (Schmitt et al., 1989). Summation of Eqs. (6) to (9) and substitution of (1) and (4) yields total mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{J}_v + \mathbf{J}_\ell) = 0. \tag{10}$$