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Tidal evolution of Mimas, Enceladus, and Dione

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Abstract

The tidal evolution through several resonances involving Mimas, Enceladus, and/or Dione is studied numerically with an averaged resonance model. We find that, in the Enceladus–Dione 2:1 e-Enceladus type resonance, Enceladus evolves chaotically in the future for some values of k_2/Q. Past evolution of the system is marked by temporary capture into the Enceladus–Dione 4:2 ee'-mixed resonance. We find that the free libration of the Enceladus–Dione 2:1 e-Enceladus resonance angle of 1.5° can be explained by a recent passage of the system through a secondary resonance. In simulations with passage through the secondary resonance, the system enters the current Enceladus–Dione resonance close to tidal equilibrium and thus the equilibrium value of tidal heating of $1.1(18{,}000/Q_S)$ GW applies. We find that the current anomalously large eccentricity of Mimas can be explained by passage through several past resonances. In all cases, escape from the resonance occurs by unstable growth of the libration angle, sometimes with the help of a secondary resonance. Explanation of the current eccentricity of Mimas by evolution through these resonances implies that the Q of Saturn is below 100,000. Though the eccentricity of Enceladus can be excited to moderate values by capture in the Mimas–Enceladus 3:2 e-Enceladus resonance, the libration amplitude damps and the system does not escape. Thus past occupancy of this resonance and consequent tidal heating of Enceladus is excluded. The construction of a coherent history places constraints on the allowed values of k_2/Q for the satellites.

Keywords: Enceladus; Saturn, satellites; Satellites, dynamics; Resonances, orbital; Tides

1. Introduction

Enceladus poses a problem. Cassini observed active plumes emanating from Enceladus (Porco et al., 2006). The heat emanating from the south polar terrain is estimated to be 5.8 ± 1.9 GW (Spencer et al., 2006). Radiogenic heating is estimated to account for only 0.32 GW (Porco et al., 2006). The secondary spin–orbit model (Wisdom, 2004) could account for the heating, but the system was not found to be librating (Porco et al., 2006). The only remaining source of heating is tidal heating. Tidal heating in an equilibrium configuration, one in which the eccentricities no longer change as the semimajor axes continue to tidally evolve, can be estimated independent of satellite physical properties using conservation of angular momentum and energy. Equilibrium tidal heating can account for at most 1.1 GW of heating in Enceladus (Meyer and Wisdom, 2007a).

One possibility is that Enceladus is oscillating about the tidal equilibrium (Ojakangas and Stevenson, 1986). However, Meyer and Wisdom (2007b) have shown that for the physical parameters of Enceladus, the Ojakangas and Stevenson model does not oscillate. Another possibility is that the resonance is dynamically unstable. If the system exhibited a, perhaps temporary, episode of chaotic variations in the eccentricity then the heating rate could exceed the equilibrium heating rate. We have therefore undertaken a systematic exploration of the dynamics of the saturnian satellite system, focusing on the evolution of Enceladus. In particular, we study the evolution of Enceladus and Dione in the current 2:1 e-Enceladus type mean motion resonance. We also study the evolution of Mimas and Enceladus through the several 3:2 mean motion resonances.

Though our study was primarily motivated by Enceladus, the free eccentricity of 0.02 of Mimas also poses a problem. If primordial, it should have damped in the age of the Solar System. What excited it? To address this problem we have extended our study to include the Mimas–Dione 3:1 multiplet of resonances.

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2. Model

Our model is an averaged resonance model for a mean-motion commensurability between two coplanar satellites. We include all terms, both resonant and secular, in the disturbing function up to third order in the eccentricities of both satellites. We also model the oblateness of the planet, including J_2, J_4 and J_2^2 contributions. We include tidal evolution of the orbits and tidal damping of the eccentricities. The physical parameters, such as the Qs of Saturn and the satellites, are all assumed to be constant in time. Details of the model are presented in Appendix A. We use the Bulirsch–Stoer algorithm to integrate the differential equations (Bulirsch and Stoer, 1966).

3. Equilibrium eccentricity

As a satellite system tidally evolves regularly into resonance, the eccentricity of one (or both) of the satellites grows because of the resonance interaction. As the eccentricity grows the dissipation grows with the square of the orbital eccentricity. Dissipation within a satellite tends to damp the orbital eccentricity. An equilibrium is possible: the satellites evolve deeper into the resonance, until the increase of eccentricity due to the evolution deeper into the resonance is balanced by the decrease of eccentricity due to internal dissipation.

When only the eccentricity of the interior satellite is excited the equilibrium eccentricity can be calculated (Meyer and Wisdom, 2007a):

$$e_0^2 = \frac{1}{7D_0}\left\{1 - \frac{1+m_1a_0/(m_0a_1)}{1+(m_1/m_0)\sqrt{a_1/a_0}} + \left(\frac{m_1}{m_0}\right)^2\left(\frac{a_0}{a_1}\right)^6\left[\frac{n_1}{n_0} - \frac{1+m_1a_0/(m_0a_1)}{1+(m_1/m_0)\sqrt{a_1/a_0}}\right]\right\}, \tag{1}$$

where a_i, m_i, and n_i are the semimajor axes, the masses, and the mean motions of the satellites (0 for interior, 1 for exterior), and where D_0 is a measure of the relative strength of tides in the interior satellite versus tides in Saturn:

$$D_0 = \frac{k_{2,0}}{Q_0}\frac{Q_S}{k_{2S}}\left(\frac{M_S}{m_0}\right)^2\left(\frac{R_0}{R_S}\right)^5. \tag{2}$$

Here $k_{2,0}$ and k_{2S} are the Love numbers, Q_0 and Q_S are the tidal dissipation factors, m_0 and M_S are the masses, and R_0 and R_S are the radii, of the interior satellite and Saturn, respectively. When only the eccentricity of the exterior satellite is excited then the equilibrium eccentricity is given by the same formula with the 0s and 1s interchanged.

As the equilibrium eccentricity is approached, the amplitude of libration in the resonance can either decrease or increase. It is either stable or unstable. In the case of Io in the Io–Europa 2:1 e-Io resonance, the libration amplitude damps and the equilibrium resonance configuration is stable. In the case of the evection resonance in the evolution of the Earth–Moon system, the libration amplitude grows as the equilibrium eccentricity is approached (Touma and Wisdom, 1998). This allows a natural escape from the resonance with an eccentricity near the equilibrium eccentricity. In our studies of the evolution of Mimas, Enceladus, and Dione, we found that sometimes the amplitude of libration damped and sometimes it grew, depending on the resonance and the physical parameters. Sometimes, as mentioned below, the escape from resonance is assisted by temporary capture into a secondary resonance, as occurred for Miranda (Tittemore and Wisdom, 1990).

After escape from resonance, the eccentricity decays with the timescale (Squyres et al., 1983)

$$\tau = \frac{2ma^5}{21nMR^5}\frac{Q}{k_2}, \tag{3}$$

where m is the satellite mass, a is the semimajor axis, n the mean motion, M the planet mass, R the satellite radius, Q the dissipation factor, and k_2 the satellite potential Love number. Note that the k_2/Q for the satellite affects both the equilibrium eccentricity (through the factor D_0) and the timescale for eccentricity damping.

4. Enceladus–Dione 2:1 e-Enceladus resonance—Future

Enceladus and Dione are currently in the Enceladus–Dione 2:1 e-Enceladus resonance.[1] Enceladus has a forced eccentricity of about 0.0047. The system has a free libration of about 1.5° (Sinclair, 1972). We decided to explore the future evolution of the system, with the primary goal of verifying the analytic predictions of the equilibrium eccentricity for various parameters. To our surprise, we found that the system exhibits complicated, sometimes (apparently) chaotic behavior.

The behavior we found depends on the assumed k_2/Q of Enceladus, which is unknown. So we made a systematic survey varying this parameter. We explored the range of k_2/Q between 1.8×10^{-5} to 9.4×10^{-4}. The lower bound corresponds to a Kelvin[2] $k_2 = 0.0018$ with a Q of 100. The upper bound corresponds roughly to a k_2 that is 10 times the Kelvin value with a Q of 20.

For $1.8 \times 10^{-5} < k_2/Q < 7.8 \times 10^{-5}$ the system tends toward the expected equilibrium, but as the eccentricity approaches the equilibrium eccentricity the libration amplitude increases. Eventually, the system escapes the resonance whereupon the eccentricity decays.

For $7.8 \times 10^{-5} < k_2/Q < 9.4 \times 10^{-5}$, the system exhibits an unexpected and interesting behavior. As in the previous case, the system tends toward equilibrium while the libration amplitude increases. Then the system enters a phase with large chaotic variations in the eccentricity while the resonance angle alternates between circulation and libration. Eventually the system escapes resonance and the eccentricity decays. After

[1] The resonant argument of the Enceladus–Dione 2:1 e-Enceladus resonance is $\lambda_E - 2\lambda_D + \varpi_E$, where λ_E and λ_D are the mean longitudes of Enceladus and Dione, and ϖ_E is the longitude of pericenter of Enceladus. For this resonance the eccentricity of Enceladus is excited.

[2] That is, using Kelvin's formula (Love, 1944)

$$k_2 = \frac{3/2}{1+19\mu/(2\rho g R)}$$

for the Love number of a homogeneous satellite of density ρ, radius R, surface gravity g, and rigidity μ. We chose $\mu = 4 \times 10^9\ \mathrm{N\,m^{-2}}$.