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On the shapes and spins of "rubble pile" asteroids

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We dedicate this paper to our friend and colleague Steven J. Ostro. Ostro has pioneered the field of radar astronomy, particularly in exploring near-Earth asteroids and the methods of inversion of both radar and optical data to obtain shape models of irregular bodies. Without such tools we would not have detailed "images" of asteroids and binary systems that provide the essential input for the modeling in Sections 2 and 4 of this paper. Anyone who has co-authored a paper with Steve is aware, sometimes painfully so, of his insistence on rigorous analysis, and that it is at least as important to explore and define what one *cannot* say from the data, as it is to present what one *can* say. It is in this spirit that we present Section 3 of this paper.

Keywords:
Asteroids
Asteroids, dynamics
Asteroids, rotation
Rotational dynamics
Tides, solid body

ABSTRACT

We examine the shape of a "rubble pile" asteroid as it slowly gains angular momentum by YORP torque, to the point where "landsliding" occurs. We find that it evolves to a "top" shape with constant angle of repose from the equator up to mid-latitude, closely resembling the shapes of several nearly critically spinning asteroids imaged by radar, most notably (66391) 1999 KW4 [Ostro, S.J., Margot, J.-L., Benner, L.A.M., Giorgini, J.D., Scheeres, D.J., Fahnestock, E.G., Broschart, S.B., Bellerose, J., Nolan, M.C., Magri, C., Pravec, P., Scheirich, P., Rose, R., Jurgens, R.F., De Jong, E.M., Suzuki, S., 2006. Science 314, 1276–1280]. Similar calculations for non-spinning extremely prolate or oblate "rubble piles" show that even loose rubble can sustain shapes far from fluid equilibrium, thus inferences based on fluid equilibrium are generally useless for inferring bulk properties such as density of small bodies. We also investigate the tidal effects of a binary system with a "top shape" primary spinning at near the critical limit for stability. We find that very close to the stability limit, the tide from the secondary can actually levitate loose debris from the surface and re-deposit it, in a process we call "tidal saltation." In the process, angular momentum is transferred from the primary spin to the satellite orbit, thus maintaining the equilibrium of near-critical spin as YORP continues to add angular momentum to the system. We note that this process is in fact dynamically related to the process of "shepherding" of narrow rings by neighboring satellites.

1. Introduction

In recent papers by Ostro et al. (2006) and Scheeres et al. (2006), the shape and spin model of the Asteroid (66391) 1999 KW4 is remarkable in having an almost constant surface slope, in the mid-latitude range, of about 35°, almost uniformly sloping in the direction of the equator. The explanation appears to be that 1999 KW4 is a "rubble pile" rather than a monolithic body with rock-like strength, so that as it is spun up by the YORP effect[1], the local slope increases to the critical angle for land-sliding. When that occurs material slides toward the equator and re-establishes an equilibrium with the average slope of the surface being the angle of repose, which for most dry loose material is around 35° (Neuendorf et al., 2005). The "top" shape appears to be the figure of quasi-equilibrium of a body that is spinning at a rate that is critical at the equator (gravity = centrifugal force), and has a constant slope at other latitudes, except for the poles, which would

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[1] The so-called YORP (Yarkovsky–O'Keefe–Radzievskii–Paddack) effect is a torque driven by solar radiation and re-emission asymmetry on an irregular spinning body; see Rubincam (2000) for a fuller description.

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be pointed otherwise, and the equator, where slope must pass through zero. Indeed, the detailed shape of the figure allows us to infer the critical angle of repose of the regolith. In Section 2 of this paper we calculate the static figure of constant slope to compare with the radar-derived shape of 1999 KW4.

In Section 3 we digress, using the same dynamical model, to investigate the limits on shapes of non- or slowly-spinning rubble piles, to consider whether "fluid equilibrium" shapes have any relevance at all for small cohesionless bodies.

A second remarkable feature of 1999 KW4 is the extremely regular equatorial band, which is within about 1% of cancellation of gravity by centrifugal force. We suggest that this equilibrium is established by tidal forces from the satellite moving regolith around the equator. In Section 4 of this paper, we develop the equations of motion for material near the equator in the rotating frame of the primary, including tidal force from the secondary and sliding friction as material starts to move. We present numerical integrations of such mass motion to show that it results in a sort of "tidal torque" between the primary spin and the satellite orbit that transfers angular momentum to the satellite orbit, potentially much more rapidly than solid-body tides.

We are motivated in these studies by the fact that (66391) 1999 KW4 is not unique in these characteristics, indeed, it seems to be the archetypical case of an asynchronous binary system. Many of the systems observed are found to have primaries with very low amplitude lightcurves (e.g., Pravec et al., 2006), indicating nearly circular equatorial profiles, and with spin periods very near the critical limit (Pravec and Harris, 2007). Radar observations of a number of NEA asynchronous binary systems likewise indicate primaries with very regular equatorial profiles and spins near the critical limit (e.g., Shepard et al., 2006).

2. Constant-slope shape of a critically spinning body

The equilibrium figures of uniformly rotating fluids has been well studied (e.g., Chandrasekhar, 1969). Up to a certain critical spin rate, the equilibrium figure of a homogeneous fluid is an oblate spheroid, a so-called Maclauren spheroid. At higher spin rates, an equilibrium triaxial ellipsoid figure exists, called Jacobi ellipsoids, up to a point where there is no equilibrium figure that can contain more angular momentum, and a body with still more angular momentum must fission in some manner to become a binary. These shapes are hardly relevant for small solid asteroids, even ones that are "rubble piles," with no tensile strength. The reason is that even strengthless rubble can sustain a slope against the pull of gravity, just as a sand pile does not flow like water but rather relaxes into a conical form with a rather constant slope, typically in the range of 35° or so. Thus we expect that the figure of a spinning asteroid can deviate from equilibrium (zero slope) by up to some critical angle before slumping (landsliding) occurs.

A brief digression into the subject of soil mechanics is in order. We are considering dry, loose material at pressures below the crushing strength of the constituent particles, and below the level where plastic, elastic, or fluid deformation occurs. The failure criterion for such material is best described by the so-called Mohr–Coulomb model (Holsapple, 2004, goes into considerable detail on this as applied to rubble pile asteroids). Essentially, solid rubble will remain static up to a failure criterion that corresponds to a constant ratio of shear stress to normal stress. If the only loading is due to hydrostatic pressure, then the ratio of shear to normal stress is simply the tangent of the slope angle of the surface, at depths less than the scale over which the surface slope is essentially the same. Thus, if one imagines "tipping" a rubble surface, failure occurs at a specific angle of tilt, and once a "landslide" initiates, it will continue until the slope of the rubble falls below a constant angle. Geologists define two angles relating to the slope

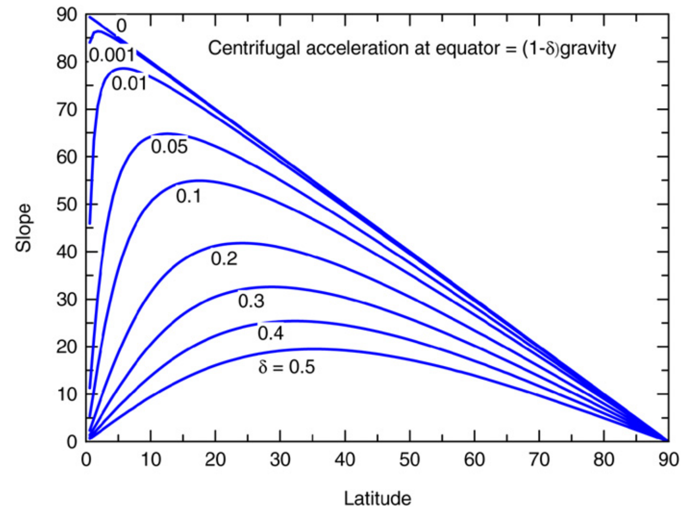

Fig. 1. Slope vs. latitude for various values of δ.

of rubble or other loose material. These angles are related to the coefficients of sliding (kinetic) friction and of static friction for the material. The condition for stability against landsliding is the maximum slope that a material such as sand, loose rock, or whatever can withstand before landsliding occurs. This angle is called the *angle of slide*, related to the coefficient of static friction: $\tan\beta_s = \mu_s$. A similar angle, the *angle of repose*, β_r, is the slope at which a landslide stops, or comes to rest. This angle is related to the coefficient of sliding (kinetic) friction: $\tan\beta_r = \mu_k$. The coefficient of sliding friction is always less than the coefficient of static friction (traction decreases once sliding starts, as a person driving in a skid discovers). For typical dry rubble on the Earth, the angle of repose is in the range of about 35°. The angle of slide is typically 5° to 10° greater (Neuendorf et al., 2005). Thus nominal values $\beta_r = 35°$ and $\beta_s = 42°$ correspond to $\mu_k = 0.7$ and $\mu_s = 0.9$.

To model the quasi-equilibrium shape of a rubble pile, we first define a relation between the gravitational acceleration at the equator, g, and the centrifugal force due to rotational spin:

$$\omega^2 R = (1-\delta)g = (1-\delta)\omega_0^2 R, \quad (1)$$

where ω is the angular frequency of rotation and R is the equatorial radius of the body. ω_0 is the critical spin frequency at which acceleration is zero; for a sphere, $\omega_0^2 = GM/R^3$, but for a non-spherical body, it will differ a bit due to gravitational harmonics. The parameter δ is chosen such that the net acceleration at the equator is δ times the gravitational acceleration, and for $\delta = 0$ acceleration vanishes. In terms of spin rate, $\delta = 1 - (\omega/\omega_0)^2$, or $\omega/\omega_0 = \sqrt{1-\delta}$. It is instructive to consider the local slope versus latitude for a sphere for various values of δ. This can be computed analytically, which we have done in Fig. 1. At the critical spin rate, local slope is just equal to the co-latitude. Note that slope begins to exceed a critical value of around 40° at a value of $\delta \approx 0.2$, which translates to a spin rate of $\omega/\omega_0 \approx 0.9$. Thus, we expect that an initially spherical "rubble pile" with a typical angle of repose would begin to experience landsliding as it were gradually spun up upon reaching about 90% of the critical spin rate ω_0. This is consistent with the calculations of Holsapple (2004). However, in the last 10% of spin-up, slopes reach extreme values, thus we expect slumping to occur. The onset should be in the mid-latitude range, around 20°–30°, and with increasing spin should progress all the way to the equator quite rapidly. Note that "downhill" is always toward the equator, so slumping will only occur at latitudes less than where slope exceeds the critical value.

In order to compute quasi-equilibrium figures of a spinning body, we wrote a fairly simple program to numerically evaluate