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Planetesimal capture in the disk instability model

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Abstract

We follow the contraction and evolution of a typical Jupiter-mass clump created by the disk instability mechanism, and compute the rate of planetesimal capture during this evolution. We show that such a clump has a slow contraction phase lasting $\sim 3 \times 10^5$ yr. By following the trajectories of planetesimals as they pass through the envelope of the protoplanet, we compute the cross-section for planetesimal capture at all stages of the protoplanet's evolution. We show that the protoplanet can capture a large fraction of the solid material in its feeding zone, which will lead to an enrichment of the protoplanet in heavy elements. The exact amount of this enrichment depends upon, but is not very sensitive to the size and random speed of the planetesimals.

Keywords: Accretion; Jupiter; Planetary formation

1. Introduction

There are nearly 200 known extrasolar planetary systems. Many of these systems contain planets of several Jupiter masses. For those few systems that transit the stellar disk, the radius can be measured, and most of these transiting planets appear to be gas giants (see, e.g., Henry et al., 2000; Udalski et al., 2002; Pont et al., 2004; Alonso et al., 2004). Currently, there are two main theories for forming such planets. The core accretion theory (Pollack et al., 1996) argues that a heavy element core is built up by the accretion of planetesimals. As the core grows, its ability to accrete gas from the surrounding disk increases. When the core is sufficiently massive, there is a rapid accretion of such gas and a giant planet is formed. The big advantage of this model is that the same basic mechanism of planetesimal accretion will form terrestrial planets, Uranus and Neptune-sized intermediate planets, and giant planets. The disadvantages are that the core mass required to form Jupiter is at the upper end of the mass estimated from interior models (Saumon and Guillot, 2004), and that the time to reach a Jupiter mass is uncomfortably close to the upper limit estimated for lifetime of the gas disk (Haisch et al., 2001). Both these objections can be ameliorated, if not entirely removed, if the accreted material does not sink entirely to the core (Pollack et al., 1996) and if the opacities are in fact lower than those estimated from interstellar grains (Podolak, 2003; Hubickyj et al., 2005).

A competing mechanism for giant planet formation is a local disk instability (Boss, 1998). This model suggests that under the right conditions an instability can form in the protoplanetary disk. This instability can lead to the creation of a self-gravitating clump of gas and dust. Such clumps can contract to form giant gaseous protoplanets (Boss, 1997, 1998). This model has several advantages, among them, that planets can be formed quickly, before the nebular gas dissipates. Calculations by Boss (2000a, 2000b) show that the unstable disk can break up into giant gaseous protoplanets in $\sim 10^3$ yr. The instability occurs over dynamical timescales (some tens of orbital periods), and the formation of clumps (future giant gaseous protoplanets) can easily occur within the estimated lifetime of most circumstellar disks (Haisch et al., 2001).

One of the problems with the disk instability model is that the planets formed by this mechanism start with a solar abundance of elements. Observations as well as theoretical models, however, indicate that Jupiter's envelope (and Saturn's as well) is enriched with heavy elements (Young, 2003;

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Saumon et al., 1995). The theoretical estimate for the mass of heavy elements in Jupiter is $\sim$20–30$M_\oplus$ (Saumon and Guillot, 2004). A solar composition planet of Jupiter's mass would be expected to have only $\sim 6M_\oplus$ of heavy elements. This means that planets created by the disk instability mechanism must have accreted this additional material later, presumably as solids. In this paper, we follow the evolution of an isolated clump and compute the planetesimal accretion rate as a function of time. The calculation is divided into three parts: the computation of the evolution of an isolated clump, the computation of the cross section for the two body interaction between the protoplanet and a planetesimal, and the actual calculation of the mass that is accreted. The details are presented in the following sections.

2. Planetary evolution code

In our calculation, we assume an isolated non-rotating clump. The initial physical parameters of the body were chosen to fit the expected initial conditions after the onset of the gravitational instability (Boss, 2002). We assume that there are no external influences on the body, such as solar heating or disk shear. We follow the evolution of this clump using a stellar evolution code developed by one of us (Kovetz).

This code, which was originally developed for stellar evolution studies, solves the standard equations of stellar evolution:

$$\frac{\partial}{\partial m}\frac{4\pi}{3}r^3 = \frac{1}{\rho}, \tag{1}$$

$$\frac{\partial p}{\partial m} = -\frac{Gm}{4\pi r^4}, \tag{2}$$

$$\frac{\partial u}{\partial t} + p\frac{\partial}{\partial t}\frac{1}{\rho} = q - \frac{\partial L}{\partial m}, \tag{3}$$

$$\frac{\partial Y_j}{\partial t} = R_j - \frac{\partial F_j}{\partial m}, \quad F_j = -\sigma_j\frac{\partial Y_j}{\partial m}, \tag{4}$$

$$\frac{\partial T}{\partial m} = \nabla\frac{\partial p}{\partial m}, \tag{5}$$

where Y_j is the number fraction of the jth species, related to the mass fraction X_j by $Y_j = X_j/A_j$, F_j is the particle flux of the jth species, determined by the corresponding coefficient of diffusion σ_j, $\nabla(r, m, L, \rho, T, Y)$ is the temperature 'gradient' $d\log T/d\log p$, determined by the mixing length recipe (MLR), and the remaining symbols are in standard notation. At the center r, the energy flux L and the F_js all vanish. The star's surface is taken to be the photosphere. Thus, the surface boundary conditions are $L = 4\pi r^2\sigma_{SB}T^4$, $F_j = 0$ and $\kappa p = Gm\tau_s/r^2$, where τ_s is the optical depth of the photosphere, which we take to be unity.

The foregoing equations are replaced by implicit difference equations, which are then solved numerically. Instead of using a fixed grid of mass points, the code determines the mass distribution by requiring the function

$$f = \left(\frac{m}{M}\right)^{2/3} - \frac{c}{\log(p_c/p_s)}\log p \tag{6}$$

to change by a constant increment between any two consecutive mass points. With c a numerical constant of order unity, the second-order difference equation $d^2 f = 0$, which has the boundary conditions $m = 0$ at the center and $m = M$ at the surface, ensures equal steps of $\log p$, except near the center, where it imposes equal steps of $m^{2/3}$.

In the absence of any nuclear (or chemical) change, the rates R_j will all vanish, but the heating source q may still be positive, e.g., when (and where) accreted planetesimals are slowed by friction.

As noted above, convection is treated in accordance with MLR. In order to avoid the difficulties connected with sudden, instantaneous mixing in convective zones, the code regards convective mixing as a diffusive process in a gas of density ρ, mean velocity v_c (supplied by MLR) and mean free path $l_c = 1.5H_p$. The artificial convective diffusion coefficient $\sigma_c = (4\pi r^2\rho)^2 v_c l_c$ (the same for all species) of course vanishes outside the convective zones. Any real diffusion, or settling, process can then be easily incorporated by adding actual diffusion coefficients.

The equation of state tables were kindly provided by D. Saumon and are based on Saumon et al. (1995), supplemented at low pressures by our own equation of state, appropriate for a weakly interacting (Debye approximation) mixture of gases. The opacity tables were kindly provided by P. Bodenheimer, based on the work of Pollack et al. (1996). They include both gas and grain opacity, the latter being based on the size distribution relevant for interstellar grains.

We were able to find an initial quasi-static model of a Jupiter mass object, similar to the initial clump in the model of Boss (2002), based on a preliminary model, also provided by Bodenheimer. Table 1 gives the initial parameters of the starting model. The pressure and temperature profiles for the initial model are shown in Fig. 1.

Fig. 2 shows evolution of the protoplanet's radius, central temperature, and central pressure with time for the first 3×10^5 yr. At this point the central temperature reaches $\sim$2000 K and the molecular hydrogen dissociates sufficiently so that the resultant energy sink triggers a rapid collapse of the body. The evolutionary track is similar to one calculated by Bodenheimer et al. (1980), for a $1M_J$ protoplanet with an initial central density of 10^{-9} g cm^{-3} and an initial central temperature of 100 K. For comparison our $1M_J$ protoplanet initially has an central density of $\sim 2 \times 10^{-8}$ g cm^{-3}, a radius of 0.5 AU, and a central temperature of $\sim$350 K. The evolution of the body was computed assuming no planetesimal capture, so that the composition of the protoplanet was constant.

Table 1
Properties of the initial model

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Radius (cm)	7.14×10^{12}
Effective temperature (K)	26.3
Central temperature (K)	3.51×10^2
Photospheric density (g cm^{-3})	1.64×10^{-11}
Central density (g cm^{-3})	2.53×10^{-8}
Photospheric pressure (dyne cm^{-2})	1.55×10^{-2}
Central pressure (dyne cm^{-2})	3.17×10^2