Journal of Atmospheric and Solar-Terrestrial Physics 140 (2016) 10–15

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Research paper

Spherical Slepian as a new method for ionospheric modeling in arctic region

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ARTICLE INFO

Article history:
Received 13 June 2015
Received in revised form
6 January 2016
Accepted 7 January 2016
Available online 29 January 2016

Keywords:
Ionosphere – Slepian
Arctic Region
GPS Data

ABSTRACT

From the perspective of the physical, chemical and biological balance in the world, the Arctic has gradually turned into an important region opening ways for new researchers and scientific expeditions. In other words, various researches have been funded in order to study this frozen frontier in details. The current study can be seen in the same milieu where researchers intend to propose a set of new base functions for modeling ionospheric in the Arctic. As such, to optimize the Spherical Harmonic (SH) functions, the spatio-spectral concentration is applied here using the Slepian theory that was developed by Simons. For modeling the ionosphere, six International GNSS Service (IGS) stations located in the northern polar region were taken into account. Two other stations were left out for assessing the accuracy of the proposed model. The adopted GPS data starts at DOY 69 (Day of Year) and ends at DOY 83 (totally 15 successive days) in 2013. Three Spherical Slepian models respectively with the maximal degrees of K=15, 20 & 25 were used. Based on the results, K=15 is the optimum degree for the proposed model. The accuracy and precision of the Slepian model are about 0.1 and 0.05 TECU, respectively (TEC Unit=1016 electron/m2). To understand the advantage of this model, it is compared with polynomial and trigonometric series which are developed using the same set of measurements. The accuracy and precision of trigonometric and polynomial models are at least 4 times worse than the Slepian one.

1. Introduction

Over the past several decades, the Arctic region because of its physical, chemical and biological significance for maintaining global balance has become a key area of research. Consequently, various researches were funded with the aim to make a detailed and deep study of the area. Jones, Stephenson (1975) and Friedman et al. (1973) are two initial studies in this respect. The first suggested a monthly median model (Jones, Stephenson, 1975) which represents the median behavior of foF2 and the other ionospheric parameters. The model is based on spherical harmonic expansion in terms of universal time and spatial coordinates. The later research suggested an empirical model of the polar ionosphere which is based upon various kinds of data including 1958 and 1964 vertical incidence ionosonde measurements, and optical and satellite observations (Friedman et al. 1973).

Analyzing the physics of atmosphere (Dethloff et al. 1996; Brekke, 2012) and the atmospheric phenomena such as auroras and scintillations (Werink et al., 2003) are some other previous efforts in this realm of research. It must be noted that the assessment of the impacts of the solar activity on ionosphere provides valuable information to the aforementioned model (Luhr, Xiong 2010). Furthermore, ionosphere is one of the most important sources of bias in Global Navigation Satellite Systems (GNSS) (Seeber, 2003). Hence, GNSS observations are now the commonly used for remote sensing of the Earth's atmosphere (Skone, 2009; Jin et al., 2014).

Estimation and prediction of ionosphere have led researchers to come up with a number of models which can be categorized into function- and grid-based models (Liu et al., 2011). The first group includes the so-called Klobuchar model whose coefficients are transmitted by GPS signals (1987), the Polynomial model developed by Komjathy (1997) and the Trigonometric series model of Georgiadiou (1994). The second group covers the Global Ionospheric Models (GIMs) which are delivered in IONEX format (Schaer et al., 1998) and ionospheric product of Satellite Based Augmentation System (SBAS) (Liu et al., 2011). Researchers of the current study found Liu et al. (2008a) interesting and referred it to further details.

Based on previous studies and applications, the aforementioned models have proved poor in performance in the polar region (Liu et al., 2011). This has also been expressed, for example, in

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http://dx.doi.org/10.1016/j.jastp.2016.01.003
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the header of the Klobuchar data file. Since ionosphere is much more irregular in the polar area, its modeling becomes more complicated task here than any other region (Liu et al., 2011) due to:

- High impact of solar activities and complicated behavior of the geomagnetic field on the ionospheric variation in poles.
- Asynchronous dimensional resolution in ionospheric model based on geodetic coordinates.

Such problems could motivate researchers to develop new models (GhoddousiFard et al., 2011; Liu et al., 2011). The Spherical Cap Harmonic (SCH) model is the most famous one for the ionospheric modeling of the Arctic region (Liu et al., 2011). Interestingly, this method was used to map the Arctic's Total Electron Content (TEC) during the most recent solar cycle for analyzing its distribution and variations at different temporal and spatial scales (Liu et al., 2014a). The method was also deployed for predicting the Arctic mean TEC on the solar cycle scale (Liu et al., 2014b).

As mentioned before, the current research is an attempt to propose a new set of base functions for the ionospheric modeling in the Arctic. For that purpose, using Slepian theory (Slepian and Pollak, 1961; Slepian, 1976), researchers could apply the spatio-spectral concentration to the Spherical Harmonic (SH) functions. It is worth to mention that this theory propounded by Simons et al., (2006) is extended to the sphere for bandlimited functions in the SHs domain. And that also results in an optimized set of base functions which are tailored to modeling ionosphere in the Arctic region. Slepian base functions were first used for modeling ionosphere in the western part of the U.S. (Sharifi and Farzaneh, 2014). The initial application of this theory presented three-dimensional problems with an improperly posed formulation and therefore, a regularization method was deployed in order to find the solution. The Slepian theory was later also applied for improving GIMs in the Arctic (Etemadfard and Hossainali, 2015).

2. Spherical Slepian Base functions

With respect to orthogonality of SHs on sphere, the thin spherical ionospheric shell can be mathematically expressed as:

$$\int_{\Omega} Y_{lm}(\theta_{IPP}, \xi_{IPP})\, Y_{l'm'}(\theta_{IPP}, \xi_{IPP})\, d\Omega = \delta_{ll'}\delta_{mm'} \tag{1}$$

Here,δ is Kronrcker delta,Y_{IPP} is the SH base function of degree/and order m as given bellow:

$$Y_{lm}(\theta_{IPP}, \xi_{IPP}) = \begin{cases} \sqrt{2}X_{lm}(\theta_{IPP})\cos(m\xi_{IPP}) & if \quad 0 < m \le l \\ X_{lm}(\theta_{IPP}) & if \quad m = 0 \\ \sqrt{2}X_{lm}(\theta_{IPP})\sin(m\xi_{IPP}) & if \quad -l \le m < 0 \end{cases} \tag{2}$$

Here $X_{lm}$is the normalized Legendre function at the Ionosphere Pierce Point (IPP).

Since Eq. (1) is valid on the surface of the whole sphere (Ω) only, application of SHs as base functions for the ionosphere modeling in a regional scale (R) distorts the concentrated signal in such a region. Orthogonalization of SHs on Ror equivalent, the optimization of concentrated signals is a problem which can be solved by the Slepian theory. According to this theory, the spatial concentration of the bandlimited function $VTEC(\theta_{IPP}, \xi_{IPP})$ is maximized in R by:

$$\lambda = \frac{\| VTEC(\theta_{IPP}, \xi_{IPP})\|_R^2}{\| VTEC(\theta_{IPP}, \xi_{IPP})\|_{\Omega}^2} = \frac{\int_R VTEC^2(\theta_{IPP}, \xi_{IPP})d\Omega}{\int_{\Omega} VTEC^2(\theta_{IPP}, \xi_{IPP})d\Omega}$$

$$= \frac{\sum_{l=0}^{K}\sum_{m=-l}^{l}\psi_{lm}\sum_{l'=0}^{K}\sum_{m'=-l'}^{l'}D_{lm,l'm'}\,\psi_{l'm'}}{\sum_{l=0}^{K}\sum_{m=-l}^{l}\psi_{lm}^2} = \max \tag{3}$$

Here, $\psi_{lm}$is the SH coefficient and $D_{lml'm'}$is (Simons et al., 2006):

$$D_{lm,l'm'} = \int_R Y_{lm}(\theta_{IPP}, \xi_{IPP})\, Y_{l'm'}(\theta_{IPP}, \xi_{IPP})\, d\Omega \tag{4}$$

Computation of **D**is necessary for producing the Slepian base functions. The next step would be the spectral decomposition of**D**which is done using the following equation for a fix order$m$,

$$\mathbf{D}^m\mathbf{A}^m = \mathbf{A}^m\mathbf{\Lambda}^m \tag{5}$$

$\mathbf{\Lambda}^m$and $\mathbf{A}^m$ are the Eigenvalues and vector matrices for order m, respectively.$\mathbf{A}^m$is used for computing Spherical Slepian functions from SHs. This leads to the equivalent of the prolate spheroidal wave functions on the sphere as follows (Albertella et al., 1999):

$$S_{\alpha}^{m}(\theta_{IPP}, \xi_{IPP}) = \sum_{l=|m|}^{K}\left(\mathbf{a}_l^m\right)^T Y_{lm}(\theta_{IPP}, \xi_{IPP}), \quad \{|m| \le \alpha \le K\} \tag{6}$$

Here, the new set of base functions $S_{\alpha}^{m}(\theta_{IPP}, \xi_{IPP})$ is called the Spherical Slepian with each identified by two indices from the others. The first index m is equivalent to the order of the corresponding SH and the second index$\alpha$is the degree of Spherical Slepian functions although it cannot be directly compared to the degree of SHs or the concept of degree in a polynomial. Eq. (6) precisely transforms the harmonic basis of the fixed order m to the corresponding Slepian (Albertella et al., 1999). As it shows, a Spherical Slepian function of the degree α is a linear combination of the surface SHs of the same order$m$. The following equation with the Least Square (LS) approach is used for computing the corresponding coefficient in the Slepian method:

$$VTEC_{Slep}(\theta_{IPP}, \xi_{IPP}) = \sum_{m=-M}^{M}\sum_{\alpha=1}^{N}\Psi_{\alpha}^{m}S_{\alpha}^{m}(\theta_{IPP}, \xi_{IPP}) \tag{7}$$

Here, M and N are the maximum order and the degree of Spherical Slepian functions, respectively.

3. Numerical results

After the introduction of a test area, IGS stations which efficiently contribute in the development of the desired model are selected based on the IPPs distribution. Then, assuming different maximal degrees for the Slepian model, three ionospheric models are developed by GPS measurements. After conducting the accuracy and precision analysis of developed models, an optimum model is suggested for this research. To understand the advantage of this model, it is compared with polynomial and trigonometric series which are developed using the same set of measurements.

The test area can be defined as (66° 33′N) of the Arctic Circle, the approximate limit of the midnight sun and the polar night (Smithson et al. 2002). IGS has several GPS/GNSS stations in this area and to decide on the required number of GPS stations, their contributions have been analyzed. To this end, IPPs' positions were computed at each station and for every satellite during the course of this research. In other words, the number of IPPs located in the Arctic Circle were counted and compared for various stations which can be seen in Fig. 1.

As the above figure shows, the IGS stations located in Arctic region can be divided into two groups: the first includes those stations with a daily contribution of 50% or more in the test area