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Knee model: Comparison between heuristic and rigorous solutions for the Schumann resonance problem

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ABSTRACT

Rapid development of computers allows for application of the direct numerical solution of the global electromagnetic resonance problem in the Earth-ionosphere cavity. Direct numerical solutions exploit the cavity models with the given conductivity profile of atmosphere such as exponential or the knee profiles. These profiles are usually derived from the knee model by Mushtak and Williams (2002) developed for obtaining the realistic ELF propagation constant. It is usually forgotten that profiles of the knee model are only a convenient approximate interpretation for the heuristic relations used in computations. We demonstrate that the rigorous full wave solution of the electromagnetic problem for such profiles deviates from that obtained in the knee model. Therefore the direct numerical solutions must also depart from the heuristic one. We evaluate deviations of the heuristic knee model data from those pertinent to equivalent profile of atmospheric conductivity.

1. Introduction

Owing to rapid development of computing resources, the direct modeling of radio propagation in the Earth–ionosphere cavity is widely used in the studies of global electromagnetic resonance (e.g. Kirillov, 1993, 1996, 1998; Kirillov et al., 1997; Kirillov and Kopeykin, 2002; Hayakawa and Otsuyama, 2002; Otsuyama et al., 2003; Pechony and Price, 2004; Pechony, 2007; Ando et al., 2005; Morente et al., 2003; Yang and Pasko, 2005, 2007; Yang et al., 2006; Molina-Cuberos et al., 2006; Toledo-Redondo et al., 2010, 2013). The Maxwell's equations are solved numerically in this case. Typically, modern versions are used of the two-dimensional transmission line and relevant telegraph equations (regarded as 2DTU) or the grid methods. The most popular among the latter is the Finite Difference in Time Domain (FDTD) technique. A direct numerical solution demands enormous amount of operations, but its crucial advantage is in the flexibility. It allows for numerical simulation of the real Earth–ionosphere cavity with all its features, including the global ionospheric irregularities such as polar non-uniformity or the day–night asymmetry.

The FDTD technique became especially popular, so much that it is included into the MATLAB software. Typically, the electrical properties of ionospheric plasma are described by either an exponential vertical conductivity profile or a bended-knee profile (bended-knee model). These models were developed to obtain a realistic frequency dependence of the ELF propagation constant $\nu(f)$. The propagation constant is used in the classical solution of the Schumann resonance problem in the form of zonal harmonics series representation (ZHSR) (e.g. Nickolaenko and Hayakawa, 2002, 2014). It turns out that the FDTD solution deviates from that obtained by ZHSR in the knee model in spite of the application of the same field source and the "same" vertical conductivity profile $\sigma(h)$.

The ELF radio propagation in a uniform Earth–ionosphere cavity has been investigated for a long time. Field representations in the form of zonal harmonic series are given in the literature for both the frequency and the time domains (see e.g. Nickolaenko and Hayakawa, 2002, 2014). Knowledge of the complex radio propagation constant $\nu(f)$ is sufficient for computing the Schumann resonance fields. The $\nu(f)$ function is determined by the global properties of the lower ionosphere. However, owing to poor knowledge of the plasma parameters at altitudes from 40 to 100 km, the propagation constant cannot be derived from the ionospheric data, and the heuristic $\nu(f)$ models are used instead. The majority of these is based on the experimental observations. Ishaq and Jones (1977) suggested the most reliable model, which is a nonlinear function of frequency. There are also other models

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(Nickolaenko and Hayakawa, 2002, 2014). The formal solution of the inverse electromagnetic problem is still due: finding the profile of atmospheric conductivity $\sigma(h)$ from the $\nu(f)$ dependence based on the Schumann resonance records.

Knowledge of the vertical profile of atmosphere conductivity is necessary for investigating the possible electrical activity on other planets. Electric activity might be a source for the global electromagnetic resonance, and therefore an estimate is desirable of the expected resonant frequencies for the known parameters of planetary atmosphere (see Nickolaenko and Rabinowicz, 1982, 1987; Sentman, 1990a; Pechony and Price, 2004; Yang et al., 2006; Molina-Cuberos et al., 2006 and references therein). The height profile also becomes necessary when one wants to evaluate the impact of various disturbances in the lower ionosphere on the $\nu(f)$ dependence and, consequently, on the resonance pattern. These problems are related to the seismic activity and the space weather (Nickolaenko and Hayakawa, 2002, 2014).

Determining the $\nu(f)$ function is difficult, which is relevant to a particular profile $\sigma(h)$. We disregard the cases when unrealistic conductivity profiles allow for formal solving the electromagnetic problem in certain special functions: these cases were listed in Bliokh et al. (1980). The horizontally uniform layered atmosphere is assumed in the general formulation of the problem to be an isotropic medium, so that the $\sigma(h)$ profile is introduced as a set of thin vertically uniform layers. The rigorous solution of such a problem is multi-parametric. It is necessary to write the solutions of the wave equation for each of N layers and satisfy the $2N$ boundary conditions. Thus, a system of $2N$ linear equations appears for the wave transition and reflection coefficients at each layer (Wait, 1970). This problem can be reformulated to the first order differential equation for the surface impedance of the field. The equation itself becomes nonlinear, and the solution may be built only numerically. The approach is well known, which is regarded as the Full Wave Solution (FWS) (see Wait, 1970; Hynninen and Galuk, 1972; Bliokh et al., 1977). Application of FWS is associated with performing time-consuming calculations, although less massive as in the FDTD technique.

Greifinger and Greifinger (1978) suggested the approximate expressions for the propagation constant ν at a fixed ELF frequency in terms of the exponential conductivity profile $\sigma(h)$. The exponential conductivity profiles were used in the very low frequency band (VLF, 3–30 kHz) long before the Greifinger's publication (see e.g. Wait and Spies, 1964). However, as Greifinger and Greifinger (1978) have demonstrated, application of an exponential profile is different when we turn to the extremely low frequencies (ELF, 3–3000 Hz). They proposed approximate relations for computing the complex propagation constant ν involving the characteristic "electric" and "magnetic" heights together with the relevant height scales. The lower characteristic (electric) height is derived at the given frequency from the equality of the conductivity and the displacement currents. At the fixed frequency f the height h_E corresponds to the condition:

$$\sigma(h_E) = \sigma_E = 2\pi \cdot f \cdot \varepsilon_0, \tag{1}$$

where $\omega = 2\pi \cdot f$ is the circular frequency and $\varepsilon_0 = 8.859 \cdot 10^{-12}$ F/m is the permittivity of vacuum.

After finding the electric height, one turns to the upper, magnetic height h_M, where the wavelength in the plasma at the given frequency is equal to the local scale height ζ_M:

$$\sigma(h_M) = \sigma_M = [4\mu_0 \cdot \omega \cdot {\zeta_M}^2]^{-1}, \tag{2}$$

here μ_0 is the permeability of vacuum and ζ_M is the scale height of profile in the vicinity of upper characteristic height.

With the help of two characteristic heights h_E and h_M and two height scales ζ_E and ζ_M of the classical profile by Cole and Pierce (1965) the values of the propagation constant were obtained by the measurements of ELF radio signals transmitted by the Wisconsin Test Facility. Thus the Greifinger and Greifinger (1978) model proved to be convenient and rather efficient. This is why a desire emerged to adapt it to the calculation of Schumann resonance parameters. However, there were two obstacles to overcome. The first was the fact that formulas by Greifinger and Greifinger (1978) were obtained for the flat Earth–ionosphere duct. The global resonance is possible only in a spherical cavity and at the particular frequencies when the radio waves have traveled around the planet meet in phase. Thus, spherical geometry is a requisite feature. This first obstacle was overcome by demonstrating that the formulas derived in the flat cavity are also held in the spherical geometry, provided that the signal frequency exceeds a few hertz.

The second problem is that unlike the ELF radio transmissions, the natural signals of global electromagnetic resonance cover a broad band approximately a decade. So, the second obstacle was a frustrating prospect of multiple $\sigma(h)$ plots for different values of signal frequency for finding graphically the new characteristic heights and height scales. This difficulty was overcome by deriving formulas for the electric and magnetic height as functions of frequency. For this purpose the reference height and reference frequency were introduced. All this has been done in the works by Nickolaenko and Rabinowicz (1982, 1987) devoted to estimates of feasible global resonance on other planets of Solar system. The Earth-ionosphere cavity acted in these papers as a test for assessing the accuracy of the approach. Later, similar formulas were published by Sentman (1990a, b) and Fullekrug (2000).

Advantage of approximate solutions for the propagation constant relevant to the exponential conductivity profile does not lie only in its simplicity. In addition, the approach allows for reasonable interpretation of observations in terms of rather realistic parameters of the lower ionosphere.

Further development of the approach was associated with elaboration of more sophisticated model profiles. In particular, the knee $\sigma(h)$ profiles were suggested with a bend (or kink) at an altitude between 50 and 60 km. This is the region where the conductivity switches from ionic conductivity dominating below ~50 km to a more rapidly varying electronic conductivity dominating above ~60 km (see Kirillov, 1993, 1996, 1998; Kirillov et al., 1997; Kirillov and Kopeykin, 2002; Mushtak and Williams, 2002; Pechony and Price 2004; Pechony 2007; Greifinger et al., 2007). In these works a method was suggested for determining the $\nu(f)$ propagation constant. An alternative is obtaining the effective R, L, C parameters of the cells in artificial transmission line used in the two-dimensional telegraph equations (2DTU).

A set of heuristic knee models was suggested by Pechony and Price (2004) and Pechony (2007). However, it was not emphasized that such an efficient and rather convenient approach only approximately matches the results obtained in the rigorous solution for the actual conductivity profile $\sigma(h)$. In other words, if we use the real parts of complex characteristic heights of the knee model together with the relevant scale heights for constructing the real function $\sigma(h)$ and find the complex propagation constant $\nu(f)$ from the rigorous full wave solution, the result will deviate from that based on the knee model equations. These deviations were demonstrated for the exponential profile by Jones and Knott (1999, 2003). In order to do this, the expected resonance frequencies and the Q-factors were estimated. It has been shown that the results of the exponential model deviate from the FWS for the Schumann resonance. That is, the resonant frequencies remained almost unchanged (deviations ranged between 0.15% and 1.2%), while the Q-factors or the wave attenuation departed by more than 10%.

Similarly to the exponential profile, the popular knee models remain a convenient procedure for obtaining the heuristic $\nu(f)$