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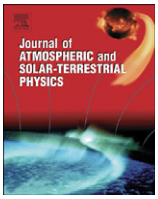

Wavelet analysis of lightning return stroke

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ABSTRACT

In this investigation, time series consisting of electric field waveforms of 15 positive return strokes and 10 negative return strokes were analyzed. The data came from a summer thunderstorm in March in a range of about 200 km around São José dos Campos, São Paulo. The electric field recording system consisted of a flat plate antenna with a decay time constant of 260 μs and a sample rate of 800,000 samples per second. The bandwidth observed was up to 100 kHz and the recording system was synchronized with GPS time and located at São José dos Campos. Wavelet analysis of the electric field waveforms was done in order to investigate the behavior of the return stroke spectrum in time. The return stroke was suggested to be divided into two portions: initial stage and overshoot (for far return stroke) and initial stage and ramp (for close return stroke). The return stroke power spectrum was observed to be distributed in a frequency range with the peak value also distributed in a fraction of this range. Power peaks for ramps are stronger than power peaks for initial stage and overshoot. Finally, it was observed how powerful the wavelet is in the analysis of lightning.

1. Introduction

1.1. Lightning spectrum observations

A cloud-to-ground lightning is a high-transient electrical discharge involving a thundercloud and ground (Miranda et al., 2003). It is composed of many processes that emit radiation in a large spectrum varying from VLF up to VHF. One of these discharges is the return stroke (or stroke for short), which makes a naked eye visible connection between cloud and ground. It is known from the literature that the stroke has the most powerful energy content in the whole lightning. Observation of the lightning radiation is important both scientifically, to understand the physics of the discharge, and in practice, to define the electromagnetic threat to electronic systems. Knowledge about the frequency spectrum is also important once it is known to be related to electric field waveform used to locate lightning and analyze its properties. For example, it is well known that the high-frequency content is attenuated due to propagation effects resulting in attenuation of electric field waveform amplitude (Uman et al., 1976; Lin et al., 1979; Weidman and Krider, 1980, 1984; Cooray and Lundquist, 1983). To understand lightning some studies in the frequency domain have been done (e.g. Serhan et al., 1980; Weidman et al., 1981; Weidman and Krider, 1986). From integration of these results we can see that the electric field spectrum of the return stroke has a maximum between 1 and 10 kHz, for first strokes at distances of about 50 km from the observer (Uman, 1987). To study the lightning spectrum, two techniques have traditionally been employed (Willet et al., 1989, 1990): (1) narrow band receivers (NBRs) tuned to different frequencies have been used to direct measurements of average and peak spectral amplitudes in frequency domain. Many investigations with this technique have been made in RF and HF (see reviews by Le Vine, 1987; Nanevicz et al., 1987), in which an increase in the pulses rates with increasing frequency was found. Whereas pulses corresponding to large and abrupt

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changes in the electrostatic field (return strokes, K changes, etc.) are of dominant occurrence in observations at lower frequencies, they are less observable in RF and HF frequencies. One disadvantage of this technique is that for HF or higher frequencies the measurements show a considerable scatter and they usually cannot be related to specific lightning processes (Willet et al., 1990). (2) Due to advances in wide band waveforms digitizers (WWDs) and signal processors which made possible electric and magnetic field records in time domain with minimal distortion, several investigators have obtained lightning radiation spectra from lightning radiation waveform through the fast Fourier transform tool. The advantage of this technique is that a spectrum can be associated with a particular lightning process according to the shape of the waveform (Willet et al., 1989, 1990). The lightning spectrum as a whole for both techniques are in good agreement in frequencies below 1 MHz. Above this frequency there is a substantial increase in the scatter of the NBR data, an unresolved disagreement. The possible causes for this disagreement can be superposition of signals from multiple sources that arrives almost simultaneously at the receiver or other lightning processes, not previously detected and studied by the WWD technique, which can be significantly stronger sources of HF frequency than return strokes. Evidence for such a strong radiation source at HF frequency has been given by Le Vine (1980) (Willet et al., 1989). Using the WWD technique Willet et al. (1989) observed unusual narrow positive bipolar pulses as being the strongest sources of HF radiation. With the same technique and in the 0.2–20 MHz band, Willet et al. (1990) observed no significant differences between the first and subsequent stroke spectra, stepped and dart-stepped leaders spectra. Willet et al. (1995) through the WWD technique found that first-stroke waveforms contain about 18 db more spectral power than subsequent-stroke waveforms, in the interval from 500 kHz to at least 7 MHz, for at least 13 μs after the main peak.

Although WWD presents advantage on the NBR technique, WWD also presents the disadvantage to study the spectrum in frequency space only, once it uses Fourier analysis, a powerful tool that permits to locate properties in frequency space only. To my knowledge, up to now investigators have pointed the attention in the observation of the lightning spectrum in frequency space only, assuming it to be a stationary process. A revolution in the signal analysis named Wavelet theory has appeared in which permits to locate properties simultaneously in frequency and time space (frequency–time space). As it will be seen ahead, it is a powerful tool that allows the Stroke spectrum to be seen as a dynamical variable. The purpose of this paper is a wavelet analysis of electric field from lightning that occurred in Brazil in March (2003).

1.2. Wavelet theory

Fourier analysis is an efficient tool to analyze series which contains stationary power spectrum. For series with non-stationary power spectrum, which are the most in nature (Bolzan, 2004), wavelet is indicated to spectral analysis. Wavelet transforms were developed in 1980s by Morlet and they have been used for numerous studies. Many applications of wavelets in geophysics are cited by Kumar and Foufoula-Georgiou (1997) and Torrence and Compo (1998). The latter work is a good option that provides an easy-to-use wavelet analysis toolkit to beginners, including statistical significance testing. One advantage of wavelet analysis on the Fourier analysis is that the wavelet transform allows decomposition of a signal in time–frequency space. So one is able to determine which are the dominant modes of variability and how these modes vary in time (Torrence and Compo, 1998).

While Fourier analysis is based on an expansion of a signal in a base of orthogonal and infinite functions sine and cosine, wavelet analysis is based on an expansion of the signal in a space whose base can be orthogonal or non-orthogonal functions but finite functions. These functions are called *Wavelets*. In the wavelet analysis the *Wavelet* term refer itself to a set of functions with limited waveform (non infinite) and which are expanded ($\psi(at)$) and translated ($\psi(t+b)$) versions of a wavelet $\psi(t)$. The translated and expanded versions are called daughter wavelets, while $\psi(t)$ is the mother wavelet. Generally we have

$$\psi_{a,b}(t) = \frac{1}{\sqrt{2}}\psi\left(\frac{t-b}{a}\right) \tag{1}$$

where $1/\sqrt{2}$ is a normalization factor. The wavelet transform of a time series $x(t)$ is

$$X(a,b) = \int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)\mathrm{d}t \tag{2}$$

where b is the translation parameter in time space and a is the dilatation parameter also known as Scale, similar to frequencies components of the signal.

There are two types of wavelets: discontinuous (discrete) and continuous wavelets, each of them with the own applications. The discrete wavelets are used for decomposing and filtering a signal, while continuous wavelets are usually used for power spectrum analysis in time–frequency space (Bolzan, 2004). The wavelets also can be classified as orthogonal or non-orthogonal. The term "wavelet basis" refers only to orthogonal set of functions and implies the use of the discrete wavelet transform. Non-orthogonal set of functions can be used with either the discrete or the continuous wavelet transforms (Torrence and Compo, 1998). One of the most common discrete wavelet is the Haar wavelet (Fig. 1, a). It is a category of wavelet used to analyze time series with length (L) of the order of power of 2 ($L = 2^n$), where n is the number of points in the sample. The Morlet, Paul and Dog wavelets are the most common continuous wavelets. Analytical expressions for these wavelets are presented in Torrence and Compo (1998) and Fig. 1 (b–d) shows some examples of these wavelets. The wavelets can be real or complex. The Morlet and Paul wavelets are complex and the Dog is real. Complex wavelets are ideal for capturing oscillatory behavior once they return information about