Journal of Atmospheric and Solar-Terrestrial Physics 68 (2006) 1321–1329

Journal of ATMOSPHERIC AND SOLAR-TERRESTRIAL PHYSICS

www.elsevier.com/locate/jastp

Piecewise continuous distribution function method: Fluid equations and wave disturbances at stratified gas

D.A. Vereshchagin[a], S.B. Leble[b,*]

[a]Theoretical Physics Department, Kaliningrad State University, 236041 Kaliningrad, Al. Nevsky str. 14, Russian Federation

[b]Theoretical Physics and Mathematical Methods Department, Technical University of Gdansk, ul, Narutowicza 11/12, Gdansk, Poland

Received 27 April 2005; received in revised form 20 March 2006; accepted 2 May 2006

Devoted to D.A. Vereshchagin memory

Available online 15 June 2006

Abstract

Wave disturbances of a stratified gas are studied. The description is built on a basis of the Bhatnagar–Gross–Krook (BGK) kinetic equation which has reduced down the level of fluid mechanics. The double momenta set is introduced inside a scheme of iterations of the equations operators, dividing the velocity space along and opposite gravity field directions. At both half-spaces the local equilibrium is supposed. As a result the momenta system is derived. It reproduces Navier–Stokes and Barnett equations at the first and second order at high collision frequencies. The homogeneous background limit gives the known results obtained by direct kinetics applications of Loyalka and Cheng as well as the recent higher momentum fluid mechanics results of Chen, Rao and Spiegel. The ground state declines from exponential at the Knudsen regime. The WBK solutions for ultrasound in exponentially stratified medium are constructed in explicit form, evaluated and plotted.

Keywords: Fluid dynamics; Rarefied gas; Stratification; Knudsen regime; Ultrasound; WBK solution

1. Introduction

There are gas dynamics problems the solution of which needs a theoretical basis going out of traditional Navier–Stokes hydrodynamics. It is connected with a break of the condition: $Kn = l/L \ll 1$, where Kn is the Knudsen number, l the free particle path, and L the characteristic scale of non-homogeneity of a problem. Perhaps, the first work, in which a wave disturbance in a gas was investigated from the point of view of more general kinetic approach, was the work of Wang Chang and Uhlenbeck (1952). The authors have offered a method of a dispersion relation construction for a homogeneous gas directly from Boltzmann equation.

The further experimental and theoretical researches of Meyer and Sessler (1957), Greenspan (1965), Foch and Ford Jr. (1970), Buckner and Ferziger (1966a,b), Sirovich and Thurber (1961, 1963, 1965, 1967), Thomas and Siewert (1979), Loyalka and Cheng (1979), Cheng and Loyalka (1981), Monchik (1964) and Banankhah and Loyalka (1987) of sound propagation in a homogeneous gas have shown that at Knudsen numbers of the unit order the waves behavior considerably

*Corresponding author. Tel.: +48 58 3471295; fax: +48 58 3472821.

E-mail addresses: leble@mif.pg.gda.pl, leble@mifgate.pg.gda.pl (S.B. Leble).

1364-6826/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.jastp.2006.05.003

differs from ones predicted on a basis of Navier–Stokes equations. These researches revealed two essential features: first, the perturbations keep wave properties at more large values of Kn than it could be assumed on a basis of the classical hydrodynamic description. Secondly, at $Kn \geqslant 1$ such concepts as the wave vector and the frequency of a wave become ill-determined. May be the most adequate results that reproduce experiments of Meyer and Sessler (1957) and Greenspan (1965) almost in all the Kn number range were obtained in Banankhah and Loyalka (1987). It is more difficult to explore the case when the Knudsen number is non-uniform in space or in time and a disturbance passes the Knudsen regime area. The statement and the solution of such problems should definitely be based on kinetic equations or their advanced model analogues (Shchekin et al., 1990).

Quite recently interest to the problems has grown again in connection with general fluid mechanics development (Leble and Vereshchagin, 1993; Vereshchagin and Leble, 1993, 1996; Vereshchagin et al., 1993; Chen et al., 2000, 2001). It was pushed by more deep understanding of perturbation theory (so-called non-singular perturbations), see, e.g., Leble (1991).

In his paper we consider a gas medium, stratified exponentially in gravity field, directed along the z-axis. It means that the Knudsen number also depends on z: $Kn(z)$. We continue to develop the method (Vereshchagin and Leble, 1993, 1996) that goes up to the pioneering paper of Lees (1965). The construction of analytical solutions of the model kinetic equation Bhatnagar–Gross–Krook (BGK) (Gross and Jackson, 1959) is extracted via separate representation of the distribution function as the local equilibrium one but with different momenta sets at positive and negative velocity component v_z half-spaces.

Thus, the set of parameters determining a state of the gas increases twice. Such number of parameters of the distribution function (6) results in that the distribution deviates from local equilibrium and accordingly widen hydrodynamics. In the range of small Knudsen numbers $l \ll L$ we have $\hat{M}_n^+ = \hat{M}_n^-$ and the distribution function (6) tends to local equilibrium one, giving a solution at the Navier–Stokes hydrodynamical regime. For big Knudsen numbers the same formula (6) gives solutions of so-called collisionless problems. Similar ideas have been successfully applied to a series of problems. For example, in papers of Lees (1965), Liu and Lees (1961) and Yang and Lees (1956), Shidlovskij (1965), a method of discontinuous distribution functions was used for the description of a flat and cylindrical (neutral and plasma) flows. For a problem of a flat flow the surface of break in space of speeds was determined by the same natural condition $V_z = 0$, and in a cylindrical case $V_r = 0$, where V_z and V_r are axial and radial components of speed of particles, respectively. The problem of a disturbance launched by a pulse movement of plane (Shidlovskij, 1965) was solved similarly.

In a problem of a shock wave structure (see Shidlovskij, 1965; Mott-Smith, 1951; Nambu and Watanabe, 1984) the solution was represented as a combination of two local equilibrium functions, one of which determines the function before front of a wave, and another the tail. In a problem of condensation and evaporation of drops of any size (Sampson and Springer, 1969; Ivchenko, 1987) the break surface was determined by so-called "cone of influence", thus all particles were divided into two types: flying "from a drop" and flying "not from a drop".

In the first two sections we derive the basic equations using the iterations in the evolution operator along the idea of the non-singular perturbation method (see, e.g. Leble, 1991). Next (Section 4) we analyze the transition to a limiting

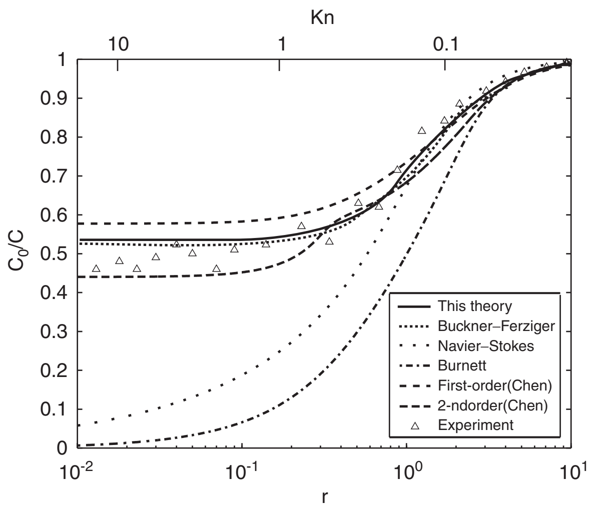

Fig. 1. The inverse non-dimensional phase velocity as a function of the inverse Knudsen number. The results of this paper are compared to Navier–Stokes, Barnett ones, the first and the second order theories of Chen et al. (2001), the results of Buckner and Ferziger (1966a,b) based on the direct solution of the Boltzmann equation (BGK-model) and the experimental data of Meyer and Sessler (1957) and Greenspan (1965).