New Astronomy 47 (2016) 42–45

Contents lists available at ScienceDirect

New Astronomy

journal homepage: www.elsevier.com/locate/newast

Investigating a suitable equation of state for an infinite system of nucleons

Niloufar Zoghi-Foumani*, Mohammad Reza Shojaei

Department of Physics, Shahrood University of Technology, Tehran Avenue, Shahrood, P. O. Box 36155-316, Iran

HIGHLIGHTS

- The average energy variation with respect to the density of a system of nucleons is presented.
- A new suitable formula is proposed for the nuclear equation of state.
- The mathematical model describes infinite symmetric/asymmetric system of protons and neutrons.

ARTICLE INFO

Article history:
Received 11 June 2015
Revised 27 January 2016
Accepted 6 February 2016
Available online 16 February 2016

Communicated by J. Silk

Keywords:
Nuclear equation of state
Infinite nuclear matter
Equilibrium state
Symmetric and asymmetric systems

ABSTRACT

In this paper, the average energy variation with respect to the density of a system of nucleons is theoretically studied. A new formula is proposed for the nuclear equation of state. This formula is related to an infinite system of protons and neutrons with relatively small thermal excitation. It is shown that the presented formulation for the nuclear equation of state reproduces the results obtained in the Skyrme–Hartree–Fock (SHF) and Relativistic Mean-Field (RMF) models of nuclear matter. It should be realized that the consistency of the obtained results for nuclear matter with the predictions of the well-known SHF and RMF models for symmetric and asymmetric system of nucleons indicates the reliability of this formulation for various types of nuclear matter in large scales such as neutron stars.

1. Introduction

The nuclear equation of state plays an important role in determining the stellar structure and evolution (Lattimer and Prakash, 2001). In addition to the astrophysical viewpoint, the nuclear equation of state helps us to describe nuclear heavy ion reactions more precisely (Li, 2002; Danielewicz et al., 2002) and specify some fundamental static properties of heavy nuclei such as binding energy and stability of neutron-rich nuclei (Todd and Piekarewicz, 2003). The nuclear equation of state is usually interpreted as the average energy of a nucleonic system per number of nucleons ($\frac{E}{A}$). This equation of state can explain the average energy per nucleon of a symmetric nuclear matter, around its equilibrium state, using two variables: temperature $T$and density ρ

$$\frac{E_{sym}}{A} = e(\rho, T), \tag{1}$$

and it can describe the average energy per nucleon of an asymmetric nuclear matter, around the condition in which the system is equilibrated, using three variables: temperature T, density ρ and δ which appears as a result of asymmetry in the number of protons and neutrons in the system

$$\frac{E_{asym}}{A} = e(\rho, T, \delta), \tag{2}$$

this variable is defined as the following:

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = \frac{\rho_n - \rho_p}{\rho}, \tag{3}$$

where ρ_n, ρ_p and ρ are the neutron, proton and total densities, respectively.

In order to investigate the pressure inside the system of nucleons, one can use the Eq. (4):

$$P = 9\rho^2 \left.\frac{\partial^2 e}{\partial \rho^2}\right|_{s=const.}, \tag{4}$$

where the pressure P is dependent to the equation of state and s is the entropy (Sosin and Kallunkathariyil, 2014; Chen et al., 2003).

When the thermal energy in a nucleonic system is low, the temperature Twill be negligible and the nuclear equation of state for the asymmetric nuclear matter around its equilibrium state will be expressed as a function of two variables: ρ and δ. In

* Corresponding author. Tel.: +98 23 32392205; fax: +98 1334773254.
E-mail addresses: niloofar.zoghi@gmail.com (N. Zoghi-Foumani), Shojaei_1151@yahoo.com (M.R. Shojaei).

http://dx.doi.org/10.1016/j.newast.2016.02.002
1384-1076/

such circumstances, one can show from Refs. Chen et al. (2003), Sosin (2010), Wiringa et al. (1988), Lattimer et al. (1991), Siemens (1970), Baym et al. (1971), Prakash et al. (1988) and Thorsson et al. (1994) that the average energy per nucleon of an infinite asymmetric system of protons and neutrons at relatively low temperatures can be written as the Eq. (5) which indicates an approximate form of the nuclear equation of state:

$$e(\rho, \delta) = e_0(\rho) + \delta^2 e_s(\rho), \tag{5}$$

where $e_0(\rho)$infers the average energy density per nucleon related to the symmetric nuclear matter with $\delta = 0$ and $e_s(\rho)$ is the symmetry energy associated to an infinite asymmetric system of nucleons. In this way, one can expand the first term in Eq. (5) around normal nuclear matter (nuclear matter at its equilibrium state) as Eq. (6):

$$e_0(\rho) = e(\rho_0) + \frac{K_0}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots, \tag{6}$$

where ρ_0 is the normal nuclear matter density (namely the density of symmetric nuclear matter at its equilibrium state with minimum energy per number of nucleons $e(\rho_0)$) and K_0 is the nuclear compressibility of normal nuclear matter (Yakhshiev and Korean, 2012; Molinelli et al., 2014) (namely the specific value of the compressibility that is taken for symmetric nuclear matter at its saturation density ρ_0) which is generally defined as (Sosin, 2010):

$$K(\rho, \delta) = 9 \left.\frac{\partial P}{\partial \rho}\right|_{s=const.}, \tag{7}$$

Furthermore, the symmetry energy in the second term of Eq. (5) can also be expanded around the normal nuclear matter density as Eq. (8):

$$e_s(\rho) = e_s(\rho_0) + \frac{L_s}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_s}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots, \tag{8}$$

where the slope L_s and the curvature K_s determine the density dependence of $e_s(\rho)$ by the following equations (Sosin, 2010; Chen et al., 2009):

$$L_s = 3\rho_0 \left.\frac{\partial e_s}{\partial \rho}\right|_{\rho=\rho_0}, \tag{9}$$

$$K_s = 9{\rho_0}^2 \left.\frac{\partial^2 e_s}{\partial \rho^2}\right|_{\rho=\rho_0}, \tag{10}$$

In order to determine the variations of the equation of state around the normal nuclear matter density, one has to specify the coefficients ρ_0, $e(\rho_0)$, K_0,$e_s(\rho_0)$, L_s and K_s. Unfortunately, the exact values of these coefficients are well determined neither in the theoretical calculations nor in experimental estimations (Sosin and Kallunkathariyil, 2014; Chen et al., 2003). For example, the experimental values for $L_s$are rare and the experimental data for various values of ρ_0 suggest $-566 \le K_s \le 34$ (Sosin, 2010; Shlomo and Youngblood, 1993; Yakhshiev and Korean, 2012). Furthermore, according to the theoretical predictions for various values of ρ_0, L_s varies from −50 MeV up to 200 MeV (Sosin, 2010, Furnstahl, 2002) and $K_s$depending on the considered model can have values between −700 MeV and +466 MeV (Sosin, 2010; Bombaci and Lombardo, 1991).

2. Proposed numerical model

In order to consider the expansions in Eqs. (6) and (8) up to second order terms (Sosin and Kallunkathariyil, 2014), one should note that these two equations are authentic only if the density of the system is sufficiently close to the density of an equilibrated system. In addition, for density $\rho \to 0$ the fundamental requirement is $e(\rho) \to 0$. Accordingly, for a symmetric system of nucleons, we propose the average energy density per nucleon in a form that can meet these necessities:

$$e_0(\rho) = \sigma_0 \rho e^{-(\eta\rho+\omega)} + c\rho, \tag{11}$$

where $\eta = 1 fm^3$ ensures the integrity of dimensions and can be interpreted as the specific volume that is the inverse density and σ_0, ω and c are constant coefficients. Using $e_0(\rho)$, the first order derivative of $e_0(\rho)$ and the second order derivative of the equation of state in symmetric condition for $\rho = \rho_0$, we can determine their values as follows:

$$\left\{ e_0(\rho_0) = \sigma_0 \rho_0 e^{-(\eta\rho_0+\omega)} + c\rho_0, \right. \tag{12}$$

$$\left\{ \begin{array}{l} \left.\dfrac{\partial e_0(\rho)}{\partial \rho}\right|_{\rho=\rho_0} = 0 \\ \left.\dfrac{\partial e_0(\rho)}{\partial \rho}\right|_{\rho=\rho_0} = \sigma_0 e^{-(\eta\rho_0+\omega)} - \sigma_0\rho_0\eta e^{-(\eta\rho_0+\omega)} + c \end{array} \right. , \tag{13}$$

$$\left\{ \begin{array}{l} \left.\dfrac{\partial^2 e_0(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0} = \dfrac{K_0}{9{\rho_0}^2} \\ \left.\dfrac{\partial^2 e_0(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0} = -2\sigma_0\eta e^{-(\eta\rho_0+\omega)} + \sigma_0\rho_0\eta^2 e^{-(\eta\rho_0+\omega)} \end{array} \right. . \tag{14}$$

Therefore, we can specify the coefficients of $e_0(\rho)$ with $e_0(\rho_0)$, K_0 and ρ_0:

$$\omega = -\left(\frac{K_0}{9e_0(\rho_0)} + 2\right), \tag{15}$$

$$\sigma_0 = \frac{1}{\eta^2{\rho_0}^3}\left(\frac{K_0}{9} + 2e_0(\rho_0)\right), \tag{16}$$

$$c = \frac{1}{\eta{\rho_0}^2}\left(\frac{K_0}{9} + e_0(\rho_0)\right). \tag{17}$$

Analogous to the equation of state of a symmetric nuclear matter, the symmetry energy in the second term of Eq. (5) can be written as:

$$e_s(\rho) = \sigma_s \rho e^{-(\eta\rho+\lambda)} + d\rho, \tag{18}$$

Where $\eta = 1 fm^3$ ensures the integrity of dimensions and its variance would also permit the properties of nuclear matter (such as skewness) to be modeled and σ_s, λ and d are constant coefficients. We can specify their values using the following equations:

$$\left\{ e_s(\rho_0) = \sigma_s \rho_0 e^{-(\eta\rho_0+\lambda)} + d\rho_0, \right. \tag{19}$$

$$\left\{ \begin{array}{l} \left.\dfrac{\partial e_s(\rho)}{\partial \rho}\right|_{\rho=\rho_0} = \dfrac{L_s}{3\rho_0} \\ \left.\dfrac{\partial e_s(\rho)}{\partial \rho}\right|_{\rho=\rho_0} = \sigma_s e^{-(\eta\rho_0+\lambda)} - \sigma_s\rho_0\eta e^{-(\eta\rho_0+\lambda)} + d \end{array} \right. , \tag{20}$$

$$\left\{ \begin{array}{l} \left.\dfrac{\partial^2 e_s(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0} = \dfrac{K_s}{9{\rho_0}^2} \\ \left.\dfrac{\partial^2 e_s(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0} = -2\sigma_s\eta e^{-(\eta\rho_0+\lambda)} + \sigma_s\rho_0\eta^2 e^{-(\eta\rho_0+\lambda)} \end{array} \right. . \tag{21}$$

Consequently, we can express the coefficients of $e_s(\rho)$ with $e_s(\rho_0)$, L_s, K_s and ρ_0 as the following relations:

$$\lambda = -\left(\frac{K_s}{9e_s(\rho_0) - 3L_s} + 2\right), \tag{22}$$