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Capillary action in a crack on the surface of asteroids with an application to 433 Eros

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HIGHLIGHTS

- The capillary action in a crack on the surface of irregular asteroids is discussed.
- The asteroid's irregular gravitational potential influences the height of the liquid in the capillary.
- Asteroid 433 Eros is taken as an example because the surface shape is irregular, elongated, and concave.
- The global maximum point for the height of the liquid in the capillary is on the concave area of asteroid 433 Eros.

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ABSTRACT

Some asteroids contain water ice, and a space mission landing on an asteroid may take liquid to the surface of the asteroid. Gas pressure is very weak on the surface of asteroids. Here we consider the capillary action in a crack on the surface of irregular asteroids. The crack is modeled as a capillary which has a fixed radius. An asteroid's irregular gravitational potential influences the height of the liquid in the capillary. The height of the liquid in the capillary on the surface of such asteroids is derived from the asteroid's irregular gravitational potential. Capillary mechanisms are expected to produce an inhomogeneous distribution of emergent liquid on the surface. This result is applied to asteroid 433 Eros, which has an irregular, elongated, and concave shape. Two cases are considered: (1) we calculate the height of the liquid in the capillary when the direction of the capillary is perpendicular to the local surface of the asteroid; (2) we calculate the height of the liquid in the capillary when the direction of the capillary is parallel to the vector from the center of mass to the surface position. The projected height in the capillary on the local surface of the asteroid seems to depend on the assumed direction of the capillary.

1. Introduction

Previous studies have claimed that Earth's water originated from asteroids (Morbidelli et al., 2000; Mottl et al., 2007; Campins et al., 2010). Kanno et al. (2003) analyzed the wavelength of infrared spectra and confirmed the presence of water ice on a D type asteroid. A meteoroid may be released from an asteroid following a collision and thus bring material to Earth (Treiman et al., 2004; Vereš et al., 2008; Ray and Misra, 2014; Patil et al., 2015). A meteoroid may also be released from Mars and bring water to Earth (Mitton, 1992). Treiman et al., (2004) studied the Serra de Magé eucrite meteorite, which presumably came from asteroid 4 Vesta, and found quartz in the meteorite; they concluded that the quartz was deposited by liquid water, and the water probably came from outside 4 Vesta. Campins et al., (2010) reported that there exist water ice on the surface of asteroid 24 Themis, and that the water ice has a widespread distribution. Comets also contain water ice. Sunshine et al., (2006) detected solid water ice on the surface of comet 9P/Tempel and pointed out that the surface deposits are loose aggregates. Taylor (2015) reported that water exists in the Eucrite meteorites which came from asteroid 4 Vesta.

We focus here on the height of the liquid surface water on the surface of an asteroid, which is related to the surface equilibrium and surface motions on the asteroid. On the surface of asteroids in the inner Solar system, liquid water can exist (Cohen and Coker, 1999; Yurimoto et al., 2014). Besides, a very transient presence of material in the liquid phase can exist in the active areas of cometary nuclei by solar heating (Miles and Faillace, 2011). Previous works have discussed the dynamics of surface equilibria on a rotating ellipsoid (Guibout and Scheeres, 2003), the hopping on

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a flat surface of a rotating ellipsoid (Bellerose and Scheeres, 2008; Bellerose et al., 2009), the contact motion and impact on the surface of asteroid 25143 Itokawa (Tardivel and Scheeres, 2014), as well as the topological classification of equilibria in the potential of asteroids (Jiang et al., 2014). Guibout and Scheeres (2003) found that the stability of surface equilibria on an ellipsoid ties to the shape of the ellipsoid. Bellerose and Scheeres (2008) discussed the dynamical behavior around the stable and unstable surface equilibria on a rotating ellipsoid to model the motion on an asteroid's surface. Jiang et al. (2014) used an accurate asteroidal shape model and discussed equilibria and motion around equilibria in the potential of four asteroids: (216) Kleopatra, (1620) Geographos, (4769) Castalia, and (6489) Golevka. Jiang (2015) finds that the equilibrium stability and the stability of periodic orbits around equilibria have some corresponding relationships. However, the height that surface water reaches on an asteroid depends on the irregular shape and gravitational potential; different surfaces produce different heights, which may lead to different friction factors and also affect the stability of equilibria.

We model a crack on the asteroids as a capillary. We study the height that a liquid can reach within this capillary that is located on the surface of irregular asteroids. The height of liquid in the capillary is constant when the position of the capillary varies over the surface of a spherical-shaped body; however, the height is time-variant when the position of the capillary varies over the surface of an irregular-shaped asteroid. The results can be applied to two areas. First, we can study the water ice distribution on asteroids; different heights of the liquid in the capillary may produce different distributions of water ice on the surface (Campins et al., 2010). Second, the height a liquid reaches can affect the electrostatic and rotational ejection of gas and dust grains (Oberc, 1997) on the surface of minor celestial bodies. Under the effect of the solar radiation pressure, the ejection will form a mini-fountain on the surface; the change in height of the liquid in the capillary causes the height and radius of the fountain envelope to vary (Oberc, 1997).

The gravitational field of asteroids influences the height a liquid can reach in a capillary; this is the case for a single asteroid, binary asteroids, and multiple asteroid systems. We present the height a liquid in a capillary can reach on the surface of an asteroid in a multiple asteroid system. Asteroid 433 Eros is taken as an example because the surface shape is irregular, elongated, and concave. The liquid's height depends on the direction of the capillary; we calculated two cases, in the first case, the capillary's direction is perpendicular to the asteroid's local surface; in the second case, the capillary's direction is parallel to the line segment from the asteroid's center of mass to the surface position. The results show that a fluid can be brought to the surface from the interior by capillary mechanisms. The process is inhomogeneous, and it is more likely that certain regions of the surface can be much more efficient than others, due to the interplay of shape, gravitational field and possible density of cracks.

2. The height of a liquid in the capillary on the surface of asteroids

Let us consider a crack on the surface of an asteroid. As we stated in the previous section, we use a capillary which has a fixed radius to model the crack. Denote r_c as the radius of the capillary. The contact angle θ is defined as the angle between the liquid's surface and the outline of the solid's contact surface. The radius of curvature of the liquid surface is denoted as R, where R is the radius of a circle which best fits the liquid surface's normal section, $R = -\frac{r_c}{\cos\theta}$. Let γ be the surface tension, then according to the Jurin's rule, the liquid height in the capillary is $h = \frac{2\gamma\cos\theta}{\rho g_a r_c}$, where ρ is the liquid density and g_a is the gravitational acceleration on the surface of the asteroid. Let α be the angle between the capillary and the direction of the gravitational force, and l be the length of liquid in the capillary, then $l = \frac{h}{\sin\alpha}$.

The gravitational field of an asteroid can be computed by assuming a shape approximated by means of a polyhedron model using observational data. The test point has the Cartesian coordinates (x, y, z), the position of the differential mass dm is (ξ, η, ς), then the vector from the test point to the differential mass is $\mathbf{r} = (\xi - x, \eta - y, \varsigma - z) = (\Delta x, \Delta y, \Delta z)$. Using the polyhedron model, the asteroid's gravitational potential (Werner, 1994; Werner and Scheeres, 1997) can be expressed by Eq. (1):

$$U = G\iiint_{Body}\frac{1}{r}\mathrm{d}m = \frac{1}{2}G\sigma\sum_{e\in edges}\mathbf{r}_e\cdot\mathbf{E}_e\cdot\mathbf{r}_e\cdot L_e - \frac{1}{2}G\sigma\sum_{f\in faces}\mathbf{r}_f\cdot\mathbf{F}_f\cdot\mathbf{r}_f\cdot\omega_f. \quad (1)$$

In addition, the gravitational force (Werner and Scheeres, 1997) is given by Eq. (2):

$$\nabla U = -G\sigma\sum_{e\in edges}\mathbf{E}_e\cdot\mathbf{r}_e\cdot L_e + G\sigma\sum_{f\in faces}\mathbf{F}_f\cdot\mathbf{r}_f\cdot\omega_f, \quad (2)$$

and the Hessian matrix of the gravitational potential is given by Eq. (3):

$$\nabla(\nabla U) = G\sigma\sum_{e\in edges}\mathbf{E}_e\cdot L_e - G\sigma\sum_{f\in faces}\mathbf{F}_f\cdot\omega_f, \quad (3)$$

where $G = 6.67\times10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant, r is the norm of $\mathbf{r}$, σ is the bulk density of the asteroid, ∇ is the gradient operator; $\mathbf{r}_e$ and are $\mathbf{r}_f$ are body-fixed vectors, $\mathbf{r}_e$ is from the test point to some fixed point on the polyhedron's edge, while $\mathbf{r}_f$ is from the test point to the point on the polyhedron's surface; $\mathbf{E}_e$ and $\mathbf{F}_f$ are edge- and face- dyads, respectively; L_e represents the integration factor between the test point and the polyhedron's edge, $L_e = \ln\frac{a+b+e}{a+b-e}$, a and b are distances between the test point and the edge's two ends, e is the edge length, ω_f represents the signed solid angle subtended by the triangle region when viewed from the test point.

Let $\boldsymbol{\omega}$ be the rotational velocity of the asteroid, and ω be the norm of the vector $\boldsymbol{\omega}$, then the body-fixed frame is defined through $\omega = \omega\mathbf{e_z}$. Then the effective potential V and its gradient (Jiang and Baoyin, 2014; Jiang, 2015; Jiang, et al. 2015) can be given by Eqs. (4) and (5):

$$V = U - M\frac{\omega^2}{2}\left(x^2 + y^2\right), \quad (4)$$

and

$$\begin{cases}\frac{\partial V(\mathbf{r})}{\partial x} = -M\omega^2 x + \frac{\partial U(\mathbf{r})}{\partial x}\\ \frac{\partial V(\mathbf{r})}{\partial y} = -M\omega^2 y + \frac{\partial U(\mathbf{r})}{\partial y}\\ \frac{\partial V(\mathbf{r})}{\partial z} = \frac{\partial U(\mathbf{r})}{\partial z}\end{cases}. \quad (5)$$

where M is the asteroid's mass. Then the gravitational acceleration of the liquid on the surface of the asteroid is given by Eq. (6):

$$g_a = \nabla V. \quad (6)$$

Assume the capillary's direction is parallel to the line segment from the asteroid's center of mass to the surface position. Substituting Eq. (6) into the expression for the height of the liquid yields Eq. (7):

$$h = \frac{2m\gamma G\sigma\cos\theta}{\rho r_c\nabla V}, \quad (7)$$

where m is the liquid mass.