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Numerical MHD codes for modeling astrophysical flows

A.V. Koldoba[a,b], G.V. Ustyugova[c], P.S. Lii[d], M.L. Comins[d], S. Dyda[e], M.M. Romanova[d,*], R.V.E. Lovelace[d,f]

[a] Moscow Institute of Physics and Technology, Dolgoprudnyy, Moscow region 141700, Russia
[b] Institute of Computer Aided Design RAS, 2nd Brestskaya st., 19/18, Moscow 123056, Russia
[c] Keldysh Institute of Applied Mathematics RAS, Miusskaya sq., 4, Moscow 125047, Russia
[d] Department of Astronomy, Cornell University, Ithaca, NY 14853, United States
[e] Department of Physics, Cornell University, Ithaca, NY 14853, United States
[f] Department of Applied and Eng. Physics, Cornell University, Ithaca, NY 14853, United States

HIGHLIGHTS

- A Godunov-type MHD code using an entropy-conserving HLLD solver has been developed.
- The entropy-conserving HLLD solver performs well for many astrophysical problems.
- Tests of the ideal MHD, viscosity and diffusivity modules are shown.
- Applications of the code to various astrophysical problems are shown.

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ABSTRACT

We describe a Godunov-type magnetohydrodynamic (MHD) code based on the Miyoshi and Kusano (2005) solver which can be used to solve various astrophysical hydrodynamic and MHD problems. The energy equation is in the form of entropy conservation. The code has been implemented on several different coordinate systems: 2.5D axisymmetric cylindrical coordinates, 2D Cartesian coordinates, 2D plane polar coordinates, and fully 3D cylindrical coordinates. Viscosity and diffusivity are implemented in the code to control the accretion rate in the disk and the rate of penetration of the disk matter through the magnetic field lines. The code has been utilized for the numerical investigations of a number of different astrophysical problems, several examples of which are shown.

1. Introduction

A number of numerical magnetohydrodynamic (MHD) codes have been developed for modeling plasma flows in astrophysics. Some of the most well-known codes are *ZEUS* (Stone and Norman, 1992a), *FLASH* (Fryxell, 2000), *PLUTO* (Mignone et al., 2007), and *ATHENA* (Stone et al., 2008; Skinner and Ostriker, 2010). A number of different numerical algorithms have also been developed for the numerical integration of the MHD equations including different approaches for the spatial and temporal approximations (Brio and Wu, 1988; Cockburn et al., 1989; Dai and Woodward, 1994a; 1994b; Ryu et al., 1995; Balsara and Spicer, 1999; Gurski, 2004; Ustyugov, 2009), and different algorithms for the approximate solution of the Riemann problem (Brio and Wu, 1988; Li, 2005; Miyoshi and Kusano, 2005; Miyoshi et al., 2010).

In this work, we describe a code developed by our group for the numerical modeling of astrophysical MHD flows. The code has been developed for use in several coordinate systems: (1) 2.5D axisymmetric cylindrical coordinates (r, z); (2) 2D polar coordinates (r, ϕ); (3) 3D cylindrical coordinates (r, ϕ, z); and (4) a Cartesian (x, y) geometry which is used to conduct tests of the ideal and non-ideal MHD modules. The difference between our code and the above-mentioned codes lies in the specifics of the astrophysical problems that we solve. Firstly, in the astrophysical regimes that we study, strong shocks (where the energy dissipation cannot be neglected) are not expected to occur. This permits the use of the entropy conservation equation instead of the full energy equation. The advantage of this approach is that the entropy conservation equation does not contain terms that differ significantly in magnitude. For example, in the energy equation, the largest terms are the gravitational energy and the kinetic energy of the azimuthal motion; these can be much larger than the internal energy and the energy of the poloidal motion near a gravitating body

* Corresponding author. Tel.: +1 607 255 6915; fax: +1 607 255 3433.
E-mail addresses: koldoba@rambler.ru (A.V. Koldoba), ustyugg@rambler.ru (G.V. Ustyugova), psl46@cornell.edu (P.S. Lii), megan.comins@gmail.edu (M.L. Comins), sd449@cornell.edu (S. Dyda), romanova@astro.cornell.edu (M.M. Romanova), lovelace@astro.cornell.edu (R.V.E. Lovelace).

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(for example, a planet or star). Additionally, in the vicinity of a magnetized star, the magnetic energy density can significantly exceed the matter pressure (and the energy-density of thermal energy), leading to significant errors when computing the energy conservation equation.

The different geometries have each been developed to solve a specific astrophysical problem and hence, there are slight differences in the algorithms across the various coordinate systems. The first version of the code to be developed was the axisymmetric 2.5D version; its aim is to model the interaction of accreting magnetized stars with the surrounding accretion disk. The 2D version in polar coordinates was developed later on to study the situation where planets orbit in a magnetized disk. Lastly, the 3D cylindrical version of the code was created to study intrinsically three dimensional problems such as bending waves in the disk or planets on inclined orbits. Each version of the code is based on the standard ideal MHD approach: the matter flow can be described with the one-fluid approximation.

In the 2D polar version of the code, the components of the velocity and magnetic field perpendicular to the plane of the flow are set to zero. Additionally, surface density is used instead of volume density. The formal structure of the MHD equations is the same but we make additional suggestions about the disk and the definitions of the (surface) magnetic field so that the conservation equations retain the same conservative form. Our codes use a Riemann solver based on methods developed by Miyoshi and Kusano (2005), modified to include the equation for the entropy balance.

The 2.5D axisymmetric cylindrical version of the code was developed to investigate astrophysical processes like disk accretion, magnetospheric accretion onto a magnetized star and outflows launched from the disk and star. Such MHD flows may be responsible for many the observed properties of young stars. In all of these processes, both the magnetic field of the star as well as magnetic fields induced in the magnetosphere of the star and in the accretion disk play an important role. It is often suggested that the angular momentum transport in the accretion disk is due to the magnetic turbulence associated with the magnetorotational instability (MRI) (e.g., Balbus and Hawley, 1991). However, modeling this instability requires high grid resolution in the disk, which is computationally expensive. In many cases, it is more practical to allow for steady accretion by incorporating viscosity terms into the code that mimic the angular momentum transport, but do not require high grid resolution. For this reason, the 2.5D version incorporates an α-type viscosity (Shakura and Sunyaev, 1973) which can be switched on or off depending on the phenomena being studied.

It is less clear how the matter in the disk interacts with magnetic field–that is, which physical processes provide efficient magnetic diffusivity in the disk. Reconnection of the magnetic field lines can provide an efficient diffusion mechanism, but the process of reconnection itself also requires some magnetic diffusivity. Since the exact mechanism is poorly understood, the magnetic diffusivity is often parameterized in the non-ideal MHD terms. This is implemented in the 2.5D version of the code as a diffusivity module, which can also be switched on or off, with the diffusivity coefficient represented in a way analogous to the $\alpha-$viscosity.

In the 2.5D axisymmetric cylindrical version of the code, we split the magnetic field into a fixed component associated with the stellar magnetic field (for example a dipole field) and a field induced by currents in the magnetosphere and disk (Tanaka, 1994). In contrast, the magnetic field in the 2D polar version is not split into a stellar and current-induced component as the code was initially developed to study the interaction of a planet with a disk threaded by a magnetic field. In order to accurately compute the strength of the stellar gravity on the planet, the grid is allowed to co-rotate with the planet. This approach was suggested by Kley (1998), albeit for a different situation.

The 3D cylindrical version of the code combines the two previous versions, enabling investigations of phenomena such as planet migration in a magnetized accretion disk, accretion onto a magnetized star as well as a variety of other problems. However, there is no diffusivity module implemented in the 3D version because the 3D instabilities which cause the magnetic field to diffuse are fully modeled.

In this work, we describe our Godunov-type codes which have been implemented on several different geometries and are designed for solving non-relativistic astrophysical MHD problems. **In Section 2 we present the full set of ideal 3D MHD equations in cylindrical coordinates.** Section 3 describes different Godunov approaches and the approach used in our codes. In Section 4, we describe the different coordinate systems in detail. In Section 5, we describe tests of the code. Section 6 shows applications of these codes for different astrophysical problems and we conclude with a summary in Section 7.

2. The governing equations of MHD in cylindrical coordinates

Here we present the full set of ideal 3D MHD equations. Viscosity and diffusivity are implemented for specific geometries and are described separately in Section 4.2.2. The continuity equation in conservative form is:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\phi)}{\partial \phi} + \frac{\partial (\rho v_z)}{\partial z} = 0. \quad (1)$$

Here $\mathbf{r} = (r, \phi, z)$ are the 3D cylindrical coordinates, ρ is the density, $\mathbf{v} = (v_r, v_\phi, v_z)$ is the velocity, and t is the time. The entropy conservation equation is:

$$\frac{\partial (\rho s)}{\partial t} + \frac{1}{r}\frac{\partial (r\rho s v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho s v_\phi)}{\partial \phi} + \frac{\partial (\rho s v_z)}{\partial z} = 0, \quad (2)$$

where $s \equiv p/\rho^\gamma$ is the entropy per unit mass and γ is the adiabatic index. The momentum equations in the r, ϕ and z directions are

$$\frac{\partial (\rho v_r)}{\partial t} + \frac{\partial}{\partial r}\left(\rho v_r^2 + Q - \frac{B_r^2}{4\pi}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\rho v_r v_\phi - \frac{B_r B_\phi}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho v_r v_z - \frac{B_r B_z}{4\pi}\right) = \frac{1}{r}\left(\rho(v_\phi^2 - v_r^2) - \frac{B_\phi^2 - B_r^2}{4\pi}\right) + \rho g_r, \quad (3)$$

$$\frac{\partial (\rho v_\phi)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2\left(\rho v_r v_\phi - \frac{B_r B_\phi}{4\pi}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\rho v_\phi^2 + Q - \frac{B_\phi^2}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho v_\phi v_z - \frac{B_\phi B_z}{4\pi}\right) = 0, \quad (4)$$

$$\frac{\partial (\rho v_z)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\left(\rho v_r v_z - \frac{B_r B_z}{4\pi}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\rho v_\phi v_z - \frac{B_\phi B_z}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho v_z^2 + Q - \frac{B_z^2}{4\pi}\right) = \rho g_z, \quad (5)$$

where $\mathbf{B} = (B_r, B_\phi, B_z)$ is the magnetic field vector, $Q = p + B^2/8\pi$ is the total pressure, and $\mathbf{g} = -\nabla\Phi$ is the net external force from the central star and planet. Lastly, the induction equations are:

$$\frac{\partial B_r}{\partial t} + \frac{1}{r}\frac{\partial}{\partial \phi}\left(v_\phi B_r - v_r B_\phi\right) + \frac{\partial}{\partial z}(v_z B_r - v_r B_z) = 0 \quad (6)$$

$$\frac{\partial B_\phi}{\partial t} + \frac{\partial}{\partial r}\left(v_r B_\phi - v_\phi B_r\right) + \frac{\partial}{\partial z}\left(v_z B_\phi - v_\phi B_z\right) = 0 \quad (7)$$