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Dynamics of charged shell with a cosmological constant

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HIGHLIGHTS

- Matching two RN de-Sitter solutions across the singular surface.
- Radial equation of motion of shell is deduced.
- The spherical N-shell model is proposed.
- This equation reduced to FRW universe with $\Lambda \neq 0$.

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ABSTRACT

Using the Darmois–Israel formalism technique, charged thin shell in the presence of a cosmological constant is constructed. An equation governing the behavior of the radial pressure across the junction surface is deduced. The cosmological constant term and the charge term slows down the collapse of matter.

The spherical N-shell model with an appropriate initial condition imitates quite well the FRW universe with $\Lambda \neq 0$.

1. Introduction

The possible existence of a cosmological constant is one of the most important challenges in high energy physics today, (Weinberg 1996). However, a surprising recent result coming from the analysis of high redshift supernovae, indicating that the universe may be accelerating now, (Cohn 1998). In fact this suggests that there is a cosmological constant which dominates the content of energy of the universe today. The gravitational collapse is one example of these extreme physical conditions where black holes seem to be formed.

The general relativistic treatment of an infinitely thin shell has been given by Israel (1966). The motion of a shell is described as a timelike hypersurface between two different given space- times. This metric junction method was generalized to include a non-vacuum metric. The compact stellar objects such as white dwarf and neutron star are formed by a period of gravitational collapse. It is interesting to consider the appropriate geometry of interior and exterior regions and determine proper junction conditions which allow the matching of these regions. Most of the problems related to gravitational collapse have been discussed by considering spherically symmetric system. The gravitational collapse of dust was first shown by Oppenheimer and Snyder (1939), the evolution of bubbles and domain walls in cosmological settings, Berezin et al. (1987), and shells around black hole solutions, Brady et al. (1991).

An interesting application to the motion of dust shell with a cosmological constant was done in Yamanaka et al. (1992) and Lake (2000). The effect of a positive cosmological constant on spherically symmetric collapse with perfect fluid has been studied by Cissoko et al. (1998). The motion of charged shell has been studied by Kuchar (1968).

The goal of this work is to extend the study of the gravitational collapse in the presence of a charge and a cosmological constant. This paper is organized as follows. In Section 2 the Darmois–Israel thin shell formalism is briefly reviewed. Match an interior RN- de-Sitter solution to an exterior RN- de-Sitter solution and the equations of motion of thin shell and the general form of these equations in N-shell are deduced in Section 3. Finally, some concluding remarks are made in Section 4. Also adopt the units such that $c = G = 1$.

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2. The Darmois–Israel formalism

Consider two distinct spacetime manifolds M_+ and M_- with metrics given by $g^{+}_{\mu\nu}(x^{\mu}_{+})$ and$S^{ij}\bar{K}_{ij} = [-T_{\mu\nu}n^{\mu}n^{\nu} - \frac{\Lambda}{8\pi}]^{+}_{-}$, in terms of independently defined coordinate systems $x^{\mu}_{\pm}$. The manifolds are bounded by hypersurfaces Σ_+ and Σ_-, respectively, with induced metrics $g^{\pm}_{ij}$. The hypersurfaces are isometric, i.e. $g^{+}_{ij}(\xi) = g^{-}_{ij}(\xi) = g_{ij}(\xi)$, in terms of the intrinsic coordinates, invariant under the isometry. A single manifold M is obtained by gluing together M_+ and M_- at their boundaries, i.e. $M = M_+ \cup M_-$, with the natural identification of the boundaries $\Sigma = \Sigma_+ = \Sigma_-$. The second fundamental forms (extrinsic curvature) associated with the two sides of the shell are:

$$K^{\pm}_{ij} = -n^{\pm}_{\gamma}\left(\frac{\partial^2 x^{\gamma}}{\partial\xi^i\partial\xi^j} + \Gamma^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right)\Big|_{\Sigma} \tag{1}$$

where $n^{\pm}_{\gamma}$ are the unit normal 4-vector to Σ in M, with $n_{\mu}n^{\mu} = 1$ and $n_{\mu}e^{\mu}_{i} = 0$. The Israel formalism requires that the normal point from M_- to M_+. For the case of a thin shell K_{ij} is not continuous across Σ, so that, the discontinuity in the second fundamental form is defined as $[K_{ij}] = K^{+}_{ij} - K^{-}_{ij}$. The Einstein equations determines the relations between, the extrinsic curvature and the three dimensional intrinsic energy momentum tensor, are given by the Lanczos equations:

$$S_{ij} = \frac{-1}{8\pi}([K_{ij}] - [K]g_{ij}) \tag{2}$$

where $[K]$ is the trace of $[K_{ij}]$ and S_{ij} is the surface stress-energy tensor on Σ.

The first contracted Gauss- Kodazzi equation or the "Hamiltonian" constraint,

$$G_{\mu\nu}n^{\mu}n^{\nu} = \frac{1}{2}(K^2 - K_{ij}K^{ij} - {}^3R), \tag{3}$$

with the Einstein equations provide the evolution identity

$$S^{ij}\bar{K}_{ij} = \left[-T_{\mu\nu}n^{\mu}n^{\nu} - \frac{\Lambda}{8\pi}\right]^{+}_{-}. \tag{4}$$

The convention, $[X] = X^+ - X^-$, and $\bar{X} = \frac{1}{2}(X^+ + X^-)$, is used.

The second contracted Gauss- Kodazzi equation or the "ADM" constraint,

$$G_{\mu\nu}e^{\mu}_{i}n^{\nu} = K^{j}_{i;j} - K_{,i} \tag{5}$$

with the Lanczos equations gives the conservation identity

$$S^{i}_{j;i} = [T_{\mu\nu}e^{\mu}_{i}n^{\nu}]^{+}_{-}. \tag{6}$$

The surface stress-energy tensor may be written in terms of the surface energy density σ, and surface pressure p as: $S^{i}_{j} = diag\cdot(-\sigma, p, p)$. For spherically symmetric thin shell, the Lanczos equations reduce to

$$\sigma = \frac{-1}{4\pi}[K^{\theta}_{\theta}] \tag{7}$$

$$p = \frac{1}{8\pi}([K^{\tau}_{\tau}] + [K^{\theta}_{\theta}]). \tag{8}$$

If the surface stress-energy terms are zero, the junction is denoted as a boundary surface. If surface stress terms are present, the junction is called a thin shell.

3. Generic dynamic of charged thin shell

The matching of two Reissner Nordstrom de-Sitter space-times of $M^{\pm}$, given by the following line elements:

$$ds^2_{\pm} = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{9}$$

with

$$f_{\pm} = 1 - \frac{2m_{\pm}}{r} + \frac{q^2_{\pm}}{r^2} - \frac{1}{3}\Lambda_{\pm}r^2 \tag{10}$$

where $m_{\pm}$, $q_{\pm}$ and $\Lambda_{\pm}$ are the gravitational mass, the charge and the cosmological constant outside and inside the shell. The suffix '+' denotes a quantity evaluated just outside the shell and '-' just inside the shell. Let the equation of the shell be $r_{\pm} = R_{\pm}(\tau)$, the history of the shell is described by the hypersurface $x^{\alpha}_{\pm} = x^{\alpha}_{\pm}(\tau, \theta, \varphi)$, in the regions $M^{\pm}$, respectively; the function $R(\tau)$ describes the time evolution of the shell. Using the Einstein field equation in an orthogonal reference frame, the stress-energy tensor components are given by

$$\rho(R) = \frac{-1}{8\pi}\left[-\frac{1}{R^2} + \frac{f_{\pm}}{R^2} + \frac{f'_{\pm}}{R}\right] \tag{11}$$

$$\mathrm{P}_r(R) = \frac{1}{8\pi}\left[\frac{1}{R^2} - \frac{f_{\pm}}{R^2} - \frac{f'_{\pm}}{R}\right] \tag{12}$$

$$\mathrm{P}_t(R) = \frac{1}{8\pi}\left[-\frac{f''_{\pm}}{2} - \frac{f'_{\pm}}{R}\right] \tag{13}$$

where $\rho(R)$ is the energy density, $\mathrm{P}_r(R)$ is the radial pressure, and $\mathrm{P}_t(R)$ is the lateral pressure measured in the orthogonal direction to the radial direction; the prime denotes a derivative with respect to R. The non-trivial components of the extrinsic curvature are given by:

$$K^{\theta\pm}_{\theta} = K^{\varphi\pm}_{\varphi} = \frac{1}{R}\sqrt{f_{\pm} + \dot{R}^2} \tag{14}$$

$$K^{\tau\pm}_{\tau} = \frac{1}{\sqrt{f_{\pm} + \dot{R}^2}}\left(\frac{m_{\pm}}{R^2} - \frac{q^2_{\pm}}{R^3} - \frac{1}{3}\Lambda_{\pm}R + \ddot{R}\right) \tag{15}$$

Therefore, the Lanczos equations are given by:

$$\sigma = \frac{-1}{4\pi R}[\sqrt{f_{\pm} + \dot{R}^2}] \tag{16}$$

$$p = \frac{1}{8\pi R}\left[\frac{1 - \frac{m_{\pm}}{R} - \frac{2}{3}\Lambda_{\pm}R^2 + \dot{R}^2 + R\ddot{R}}{\sqrt{f_{\pm} + \dot{R}^2}}\right] \tag{17}$$

Taking into account the transparency condition, $[G_{\mu\nu}U^{\mu}n^{\nu}] = 0$, the conservation identity, Eq. (6), provides the following simple relationship:

$$\frac{d}{d\tau}\sigma A + P\frac{d}{d\tau}A = 0 \tag{18}$$

where $A = 4\pi R^2$ is the area of the spheres of symmetry at constant R. In general case, the conservation identity provides the following relationship:

$$\sigma' = \frac{-2}{R}(\sigma + \mathrm{P}) + K \tag{19}$$

where K is the momentum flux given by

$$K \equiv \frac{\sigma}{R} = \frac{1}{4\pi R^2}[\sqrt{f_{\pm} + \dot{R}^2}] \tag{20}$$

This flux term vanishes in the particular case when $\mathrm{P} = -\rho$. Taking into account these relationship

$$\sigma + p = \frac{1}{8\pi R\sqrt{f_{\pm} + \dot{R}^2}}\left[-1 + \frac{3m_{\pm}}{R} - \frac{2q^2_{\pm}}{R^2} - \dot{R}^2 + R\ddot{R}\right], \tag{21}$$

$$\dot{\sigma} = \frac{-\dot{R}}{4\pi R^2\sqrt{f_{\pm} + \dot{R}^2}}\left[-1 + \frac{3m_{\pm}}{R} - \dot{R}^2 - \frac{2q^2_{\pm}}{R^2} + R\ddot{R}\right], \tag{22}$$

$$\sigma' = \frac{1}{4\pi R^2\sqrt{f_{\pm} + \dot{R}^2}}\left[1 - \frac{3m_{\pm}}{R} + \frac{2q^2_{\pm}}{R^2} + \dot{R}^2 - R\ddot{R}\right]. \tag{23}$$

For the static solutionR_0, with $\dot{R} = \ddot{R} = 0$, Eq. (23), reduced to

$$\sigma'(R_0) = \frac{1}{4\pi {R_0}^2\sqrt{f_{\pm}(R_0)}}\left[1 - \frac{3m_{\pm}}{R_0} + \frac{2q^2_{\pm}}{R^2}\right]. \tag{24}$$