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Formation of secondary critical points in thermally conducting interstellar gases

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HIGHLIGHTS

- This paper presents phase space aspects of nonlinear wave propagation in astrophysical gases.
- The existence of secondary critical points in thermally conducting gases is established under some very general conditions.
- The role of secondary critical points in structure formation in the ISM is discussed.

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ABSTRACT

The interstellar medium (ISM) is a thermally conducting gaseous medium consisting of neutral ions, and in some cases free electrons. Considering the ISM as a continuous medium, phase space analysis is applied here to investigate the nonlinear problem of heating and cooling wave front evolution in thermally conducting homogeneous interstellar gases. With an arbitrary net heating function a general criterion for determining the critical points is established which shows that for a given value of the thermal conductivity exponent α, secondary critical points emerge as a necessary feature of the gaseous medium. For a thermally conducting electron gas and for a neutral ion gas, we explicitly state the conditions for the occurrence of the secondary critical points. These critical points can either be unstable nodes, saddle points or degenerate nodes. The existence of secondary critical points is of significance in the formation of globule-like stable structures in the ISM, and in cloud formation phenomena in the interstellar gases.

1. Introduction

Formation and evolution of heating and cooling wave fronts in thermally conducting ISM is a major process for energy transfer in the medium. Such a process also occurs in other astrophysical systems as well, including solar corona heating, and galactic clusters formation (Shore, 2007). In such cases temperature perturbations generally result in causing thermal instability in the medium. This requirement is called the Field criterion (Parker, 1953; Field, 1965; see Appendix A), which allows only highly subsonic wave motion in a thermally stable medium. It follows that stable structures in the ISM cannot form unless pressure variations are negligible in the process of compressive waves formation in the gas. The resulting dynamics of the heating and cooling wave fronts is thus a highly nonlinear phenomenon. It has been subject to different numerical and analytical approaches (Balbus and Stoker, 1989; Meerson, 1996; Ibanez and Bessega, 2000). Numerical and linearization procedures have been useful in giving an analysis of how the wave fronts evolve in the gas (Ferrara and Shchekinov, 1993, Iwasaki and Inutsuka, 2012); and there have been indications that, for specific heating functions, there are steady fronts formed inside the medium (Elphick et al., 1991, 1992).

In this paper, we apply global phase space methods to explore typically nonlinear aspects of the problem. We find that the autonomous nature of the governing equations leads to a direct transformation into the phase space variables. This enables us to discuss the evolution of the heating and cooling fronts for an arbitrary given net heating function. Thus we arrive at some very general results regarding the dynamics of the compressive wave fronts in thermally conducting gases. In particular the analysis shows the existence of secondary critical points, that play an important role in the heating and cooling of thermally conducting gases. We establish here the conditions under which the secondary critical points may form in the gas, thus give a general criterion for heating functions where the secondary critical points may act as unstable

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nodes. In this analysis the heating/cooling function is not specified *a priori*, thus we make no assumption as to the form of the heating/cooling function used. This enables us to discuss the existence of critical points in a general thermally conducting gas.

In Section 2, we first give a full derivation of the governing equations, based on the mass, momentum, and energy conservation laws. Then coupled to the ideal gas equation we establish the Field criterion for thermal instability. We deduce that for the highly subsonic perturbations the governing equations reduce to a nonlinear equation, discussed here in one dimension. The results obtained here, however, apply to the higher dimensions as well. In Section 3, we use the phase space analysis for the problem for a general choice of the net heating function, and establish the existence of secondary critical points. Here we also give the general conditions under which degenerate nodes and saddle points are formed in an electron or ion gas. Lastly in Section 4 we give a discussion of the main conclusions of the study.

2. Formulation of the basic equations

We consider an initially uniformly distributed gas. Then under the continuous approximation, the basic equations are (Shore, 2007) the mass conservation equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

the momentum conservation equation,

$$\rho \frac{D\mathbf{u}}{Dt} + \nabla P = 0 \tag{2}$$

and the thermal energy conservation equation,

$$\frac{1}{1-\gamma}\frac{DP}{Dt} - \frac{\gamma}{1-\gamma}\frac{P}{\rho}\frac{D\rho}{Dt} = \nabla \cdot (\lambda \nabla T) + Q(\rho, T). \tag{3}$$

Here $Q(\rho, T)$ is the net heat gain per unit volume per unit time. For cooling rate Q^- and heating rate Q^+, we have $Q = Q^+ - Q^-$. Also here λ is a measure of thermal conductivity, and is given by the ratio of the of specific heats $\gamma = c_P/c_V$. In astrophysical gases heat gain is a function of temperature, for a fixed volume and mass of the gas.

2.1. Temperature equation for heating and cooling fronts

The compressive waves in the gaseous medium form heating and cooling fronts in the gas. We consider the motion of such hot or cold fronts in one spatial dimension. According to Field criterion (see Appendix A for a derivation), any motion of such interfaces must be highly subsonic. This means that we can neglect the velocity term in the momentum equation, hence obtain from Eq. (2) in one dimension:

$$\frac{\partial P}{\partial x} = 0. \tag{4}$$

Equivalently P is constant. It is now convenient to change to Lagrange variable $\eta(x,t)$ defined by:

$$\eta(x,t) = \int_{-\infty}^{x} \rho(x', t)\,dx'. \tag{5}$$

From the mass conservation equation (1) it then follows that

$$\left.\frac{\partial \eta}{\partial t}\right|_x = -\rho u, \tag{6}$$

u being the velocity in x-direction. This implies that $\partial/\partial x|_t = \rho\partial/\partial\eta|_t$ and $\partial/\partial t|_x = \partial/\partial t|_\eta - \rho u \partial/\partial\eta|_t$, thus the Lagrange derivative D/Dt can be expressed as:

$$\left.\frac{\partial f}{\partial t}\right|_x + u\left.\frac{\partial f}{\partial x}\right|_t = \left.\frac{\partial f}{\partial t}\right|_\eta. \tag{7}$$

Using these definitions the energy equation (3) in one dimension,

$$\frac{1}{\gamma-1}\frac{DP}{Dt} - \frac{\gamma}{\gamma-1}\frac{P}{\rho}\frac{D\rho}{Dt} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + Q \tag{8}$$

becomes

$$-\frac{\gamma}{\gamma-1}\frac{P}{\rho}\left.\frac{\partial \rho}{\partial t}\right|_\eta = \rho\frac{\partial}{\partial \eta}\left(\lambda\rho\frac{\partial T}{\partial \eta}\right) + Q. \tag{9}$$

Since in the subsonic case $P = \text{const.}$, and the equation of state is $P = (\mathcal{R}/\mu)\rho T$, it follows that $\rho \propto 1/T$. Thus both λ and Q are functions of temperature only. We re-scale the time variable as $\tau = \left((\gamma-1)P/\gamma(\mathcal{R}/\mu)^2\right)t$, and write

$$\mathcal{L}(T) = \left(\frac{\mathcal{R}/\mu}{P}\right)^2 \frac{Q(T)}{T}. \tag{10}$$

Eq. (9) then gives the governing equation for the evolution of the temperature front:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial \eta}\left(\frac{\lambda(T)}{T}\frac{\partial T}{\partial \eta}\right) + \mathcal{L}(T). \tag{11}$$

Since for a gas consisting of thermal electrons the approximate form of conductivity function is $\lambda(T) \propto T^{5/2}$ and for neutral ions $\lambda(T) \propto T^{1/2}$, we take the conductivity function proportional to T^α, where α can take value $5/2$ and $1/2$. For steadily moving wave front in η space, with velocity U, we set $\xi = \eta - Ut$. Also defining $T^\alpha = X$, Eq. (11) becomes (Elphick et al., 1991):

$$X^\beta \frac{d^2X}{d\xi^2} + U\frac{dX}{d\xi} + F(X) = 0, \tag{12}$$

where $F(X) = \alpha T^{\alpha\beta}\mathcal{L}(T)$, and $\beta = (\alpha-1)/\alpha$ which can take value $3/5$ for thermal a electron gas, and -1 for neutral ion gases. For the sake of generality we do not fix $\mathcal{L}(T)$, hence the net heating/cooling function $F(X)$; later this will enable us to precisely indicate the secondary critical points in a gas with a given form of the heating/cooling function, such as that used by Iwasaki and Inutsuka (2012).

3. Stability analysis and the existence of secondary critical points

Critical points are of particular interest for first order autonomous systems of the type

$$\frac{dy}{dt} = P(x,y), \quad \frac{dx}{dt} = Q(x,y). \tag{13}$$

These are points of stability for the system, independent of the initial conditions, toward or from which solutions tend in the asymptotic limit $t \to \pm\infty$. Such solutions may exhibit periodicity, and may be limit cycles also.

The temperature equation (12) is an autonomous equation for a given net heating function. It is equivalent to the first order system

$$\frac{dX}{d\xi} = Y, \tag{14}$$

$$\frac{dY}{d\xi} = -X^{-\beta}(UY + F(X)). \tag{15}$$

For a system of type (13), a point (x_0, y_0) in the phase space is called a critical point if $P(x_0, y_0) = 0 = Q(x_0, y_0)$.

The system (14) and (15) possesses critical points at $(X, Y) = (0, 0)$ and at $(X, Y) = (X_0, 0)$ where X_0 is the zero of the net heating function $F(X_0) = 0$. We first analyze the behavior of the solution close to the critical points, since periodic solutions