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# New upper limits on the power of general relativity from solar system dynamics

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## HIGHLIGHTS

- $R^{1+\delta}$ gravity is confronted with the solar system dynamics.
- Supplementary advances in perihelia from INPOP10a and EPM2011 ephemerides are used.
- Lense–Thirring effect and uncertainty of Sun's quadrupole are taken into account.
- Upper limits of $\delta$ we obtained are improved by about 5 orders of magnitude.

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## ABSTRACT

As a power class of generalization of Einstein's general relativity, the $R^{1+\delta}$ gravity is confronted with planetary motions in the solar system. Using the supplementary advances in the perihelia provided by current INPOP10a (IMCCE, France) and EPM2011 (IAA RAS, Russia) ephemerides, we obtain new upper limits on $\delta$ in the solar system when the Lense–Thirring effect due to the Sun's angular momentum and the uncertainty of the Sun's quadrupole moment are properly taken into account. These two factors were mostly absent in previous works dealing with $\delta$. We find that INPOP10a yields the upper limit as $\delta = (0.6 \pm 4.4) \times 10^{-24}$ and EPM2011 gives $\delta = (7.5 \pm 3.4) \times 10^{-24}$. Both of them are improved by about 5 orders of magnitude than previous result.



## 1. Introduction

Just after the birth of Einstein's general relativity (GR), attempts to generalize the theory started. These generalizations return to GR in the limit that the Ricci curvature scalar of the space–time $R$ is small. Technically, they modify the Einstein-Hilbert Lagrangian of gravitation by adding some correction terms of $R$, such as quadratic or higher order terms of $R$ which first proposed by Eddington (1923). Currently, a more general extension is to replace $R$ in the Einstein-Hilbert action with some analytic function $f(R)$ (see de Felice and Tsujikawa, 2010; Sotiriou and Faraoni, 2010 for reviews). One of motivations to consider them is to understanding the late-time acceleration of the Universe (e.g. Riess et al., 1998; Perlmutter et al., 1999).

In this work, we will focus on a power class of generalization of GR, in which the Lagrangian is proportional to $R^{1+\delta}$ and GR is reduced in the limit of $\delta \to 0$ (Buchdahl, 1970; Roxburgh, 1977; Clifton and Barrow, 2005). Clifton and Barrow (2005) intensively studied its cosmological and weak-field properties, including the behavior of the perfect-fluid Friedmann universes and isolate the physically relevant models of zero curvature, the synthesis of light elements and the perihelion precession. Among them, by assuming that Mercury follows a timelike geodesic, they found the best upper limit on $\delta$ is $7.2 \times 10^{-19}$.

Inspired by this idea, we will try to find new upper limits on the power of GR by making use of the supplementary advances of the perihelia provided by INPOP10a (IMCCE, France) (Fienga et al., 2011) and EPM2011 (IAA RAS, Russia) (Pitjeva, 2013) ephemerides. These two ephemerides were recently used in detecting gravitational effects and testing gravitational theories (e.g. Iorio and Saridakis, 2012; Iorio, 2013b, 2014a,c; Xie and Deng, 2013; Li et al., 2014; Deng and Xie, 2014). Since INPOP10a and EPM2011 are significantly improved compared with their previous versions, we expect to obtain tighter limits on $\delta$.

In Section 2, we will debrief the basics of the $R^{1+\delta}$ gravity according to Clifton and Barrow (2005) for completeness. The predicted

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perihelion precession will be connected with the data of ephemerides in Section 3. In Section 4, the supplementary advances of the perihelia provided by INPOP10a and EPM2011 will be used to obtain the limits of $\delta$ when the Lense–Thirring effect due to the Sun's angular momentum and the uncertainty of the Sun's quadrupole moment are taken into account. Conclusions and discussion will be presented in Section 5.

## 2. Basics of $R^{1+\delta}$ gravity

For completeness of this work, we only recall the basics of $R^{1+\delta}$ gravity and its primary results here (see Clifton and Barrow, 2005 for more details). Let us consider a gravitational theory based on the action (Clifton and Barrow, 2005)

$$S = \int \frac{1}{\chi}\sqrt{-g}R^{1+\delta}\mathrm{d}^4x + S_m, \quad (1)$$

where $\delta$ is a real number, $\chi$ is a constant, $g$ is the determinant of the metric tensor $g_{\mu\nu}$ and $S_m$ is the matter action. It is obvious that this action returns to the Einstein-Hilbert one in the limit $\delta \to 0$. By taking variation of the metric $g_{\mu\nu}$, we obtain the field equations (Buchdahl, 1970)

$$\delta(1-\delta^2)R^{\delta-2}R_{,\mu}R_{,\nu} - \delta(1+\delta)R^{\delta-1}R_{;\mu\nu} + (1+\delta)R^{\delta}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}RR^{\delta} - g_{\mu\nu}\delta(1-\delta^2)R^{\delta-2}R_{,\alpha}R^{,\alpha} + \delta(1+\delta)g_{\mu\nu}R^{\delta-1}\Box_g R = \frac{\chi}{2}T_{\mu\nu}, \quad (2)$$

where $\Box_g(\cdot) \equiv g^{\alpha\beta}(\cdot)_{;\alpha\beta}$. $T_{\mu\nu}$ is the energy–momentum tensor of the matter, which is defined in the usual way of Landau and Lifshitz (1975).

In order to test $R^{1+\delta}$ gravity in the weak-field limit, like standard tests of GR in the solar system, Clifton and Barrow (2005) found its static and spherically symmetric solution in the isotropic coordinates $(t, \hat{r}, \theta, \phi)$ as

$$\mathrm{d}s^2 = -A(\hat{r})\mathrm{d}t^2 + B(\hat{r})\left[\mathrm{d}\hat{r}^2 + \hat{r}^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right], \quad (3)$$

where

$$A(\hat{r}) = \hat{r}^{[2\delta(1+2\delta)]/\sqrt{PQ}}\left[1 + \frac{C}{4\hat{r}\sqrt{Q/P}}\right]^2\left[1 - \frac{C}{4\hat{r}\sqrt{Q/P}}\right]^{-[2(1+4\delta)]/Q}, \quad (4)$$

$$B(\hat{r}) = \hat{r}^{-2+2\left[(1-\delta)/\sqrt{PQ}\right]}\left[1 - \frac{C}{4\hat{r}\sqrt{Q/P}}\right]^{[4(1-\delta)/Q]}. \quad (5)$$

Here, $C$ is a constant, $P = 1 - 2\delta - 2\delta^2$ and $Q = 1 - 2\delta + 4\delta^2$.

By assuming that the spacetime of the solar system can be considered as static and spherically symmetric to first order approximation and this geometry determined by the Sun can be described by Eq. (3), Clifton and Barrow (2005) worked out the secular perihelion precession of a planet as

$$\dot{\omega} = \dot{\omega}_{\mathrm{PN}} + \dot{\omega}_{\delta}, \quad (6)$$

where $\omega$ is the argument of perihelion and dot means taking derivative against time. The first term in the right-hand side of above equation is the post-Newtonian precession of GR (Landau and Lifshitz, 1975),

$$\dot{\omega}_{\mathrm{PN}} = 3\frac{GM_{\odot}}{c^2a(1-e^2)}n, \quad (7)$$

and the second term is the leading effect of precession caused by the $R^{1+\delta}$ gravity,

$$\dot{\omega}_{\delta} = (GM_{\odot})^{-1/2}a^{-1/2}c^2e^{-2}(1-e^2)\delta. \quad (8)$$

Here, $c$ is the speed of light, $G$ is the gravitational constant, $M_{\odot}$ is the mass of the Sun, $a$ is the semi-major axis, $e$ is the eccentricity, and $n$ is the Keplerian mean motion.

## 3. Confrontation of $\dot{\omega}_{\delta}$ and data

In the case of the solar system's planets, $\dot{\omega}_{\delta}$ is closely connected with the supplementary advances of the perihelia $\dot{\omega}_{\mathrm{sup}}$ provided by modern ephemerides, such as INPOP10a (Fienga et al., 2010; Fienga et al., 2011) and EPM2011 (Pitjeva, 2013; Pitjeva and Pitjev, 2013a,b).

INPOP10a and EPM2011 were obtained by fitting the "standard model" of dynamics to observational data, where "standard model" means the Newton's law of gravity and the Einstein's GR (apart from the Lense–Thirring effect, see below for details). Therefore, the effects of the $R^{1+\delta}$ gravity were modeled neither in INPOP10a nor in EPM2011, and the parameter $\delta$ was not determined in these least-square fittings. In this sense, the results we obtain in next section may not be considered as genuine "constraints" (it would be so if one solved for them in a covariance analysis by reanalyzing the data with modified software including these effects) but as preliminary indications of acceptable values to the best of the contemporary knowledge in the field of ephemerides (see Iorio, 2014a for a further discussion).

These $\dot{\omega}_{\mathrm{sup}}$ might represent possibly mismodeled or unmodeled parts of perihelion advances according to the Newton's law and GR. They are almost all compatible with zero so that they can be used to draw bounds on quantities parametrizing unmodeled "forces" like the $R^{1+\delta}$ gravity in this case. Nonetheless, the latest results by EPM2011 (Pitjeva and Pitjev, 2013a,b) returned non-zero values for Venus and Jupiter. Although the level of their statistical significance was not too high and further investigations are required, we still take them into account in this work. In the recent past, an extra non-zero effect on Saturn's perihelion was studied (Iorio, 2009). And, about the non-zero values of the supplementary precessions of Venus and Jupiter by EPM2011 (Pitjeva and Pitjev, 2013a,b), their ratios have been recently used to test a potential deviation from GR (Iorio, 2014c).

In the construction of $\dot{\omega}_{\mathrm{sup}}$ (see Fienga et al., 2010 for details), the effects caused by the Sun's quadrupole mass moment $J_2^{\odot}$ are considered and isolated in the final results, but the perihelion shifts caused by the Lense–Thirring effect (Lense and Thirring, 1918) due to the Sun's angular momentum $S_{\odot}$ are absent. Therefore, by assuming these leading effects can be linearly added together, we can have the entire relation between $\dot{\omega}_{\delta}$ and $\dot{\omega}_{\mathrm{sup}}$ as

$$\dot{\omega}_{\mathrm{sup}} = \dot{\omega}_{\delta} + \dot{\omega}_{\mathrm{LT}} + \dot{\omega}_{\Delta J_2^{\odot}}. \quad (9)$$

Here, the Lense–Thirring term $\dot{\omega}_{\mathrm{LT}}$ is (Lense and Thirring, 1918; Iorio, 2001, 2009; Renzetti, 2013a)

$$\dot{\omega}_{\mathrm{LT}} = -\frac{6GS_{\odot}\cos i}{c^2a^3(1-e^2)^{3/2}}, \quad (10)$$

where $S_{\odot} = 1.9 \times 10^{41}$ kg m$^2$ s$^{-1}$ (Pijpers, 2003) and $i$ is the inclination of the planetary orbit to the equator of the Sun. The uncertainty

**Table 1**
Supplementary advances in the perihelia $\dot{\omega}_{\mathrm{sup}}$ given by INPOP10a and EPM2011.

| | $\dot{\omega}_{\mathrm{sup}}$ (mas cy$^{-1}$) | |
|---|---|---|
| | INPOP10a[a] | EPM2011[b] |
| Mercury | 0.4 ± 0.6 | −2.0 ±3.0 |
| Venus | 0.2 ± 1.5 | 2.6 ±1.6 |
| EMB | −0.2 ± 0.9 | – |
| Earth | – | 0.19 ± 0.19 |
| Mars | −0.04 ± 0.15 | −0.020 ± 0.037 |
| Jupiter | −41 ± 42 | 58.7 ± 28.3 |
| Saturn | 0.15 ± 0.65 | −0.32 ± 0.47 |

[a] Taken from Fienga et al. (2011).
[b] Provided by Pitjeva and Pitjev (2013a,b).