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PICsar: A 2.5D axisymmetric, relativistic, electromagnetic, Particle in Cell code with a radiation absorbing boundary

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HIGHLIGHTS

- Presented a new 2.5D axisymmetric Particle in Cell code, *PICsar*.
- *PICsar* is designed for efficiently simulating the pulsar magnetosphere.
- *PICsar* is 10,000 times faster than a 3D Cartesian code for axisymmetric sims.
- Tested *PICsar* using a specially designed series of tests.
- Simulated magnetic monopole geometry to 1% accuracy.

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ABSTRACT

We present *PICsar* – a new Particle in Cell code geared towards efficiently simulating the magnetosphere of the aligned rotator. *PICsar* is a special relativistic, electromagnetic, charge conservative code that can be used to simulate arbitrary electromagnetics problems in axisymmetry. It features stretchable body-fitted coordinates that follow the surface of a sphere, simplifying the application of boundary conditions in the case of the aligned rotator; a radiation absorbing outer boundary, which allows a steady state to be set up dynamically and maintained indefinitely from transient initial conditions; and algorithms for injection of charged particles into the simulation domain. The code is parallelized using MPI and scales well to a large number of processors. We discuss the numerical methods used in *PICsar* and present tests of the code. In particular, we show that *PICsar* can accurately and efficiently simulate the magnetosphere of the aligned monopole rotator in the force-free limit. We present simulations of the aligned dipole rotator in a forthcoming paper.

1. Introduction

The modeling of pulsar magnetospheres has a history dating back to the association of pulsars with spinning neutron stars. One of the early magnetospheric models that remains relevant today is that of Goldreich and Julian (1969), who studied a magnetosphere filled with plasma in the special case when the neutron star rotational and magnetic axes are aligned (aligned dipole rotator). The Goldreich and Julian (1969) model assumes a"force-free" magnetosphere, which means that electromagnetic fields dominate particle inertia so that the Lorentz force law for the plasma is $\rho\boldsymbol{E} + c^{-1}\boldsymbol{J}\times\boldsymbol{B} = 0$, where ρ is the charge density, $\boldsymbol{J}$ is the current, and c is the speed of light. The force-free condition implies that $\boldsymbol{E}\cdot\boldsymbol{B} = 0$, so there is enough plasma to short out the component of the electric field along magnetic field lines. The force-free approximation can be viewed as a limiting case of relativistic magnetohydrodynamics (MHD) when particle pressure and inertia are negligible (Komissarov, 2002).

The structure of the pulsar magnetosphere has been simulated using force-free and MHD simulations for both the aligned rotator (Contopoulos et al., 1999; Gruzinov, 2005; Timokhin, 2006; McKinney, 2006; Komissarov, 2006), and for arbitrary inclinations of the spin and magnetic axes (Spitkovsky, 2006; Contopoulos and Kalapotharakos, 2010; Kalapotharakos et al., 2012; Tchekhovskoy et al., 2013). These simulations were extended to include a finite resistivity by Li et al. (2012), Kalapotharakos et al. (2012), which allowed the authors to capture a spectrum of solutions between the force-free and vacuum limits. One of the observational impetuses for more accurate solutions of magnetospheric structure is the modeling of pulsar lightcurves (Bai and Spitkovsky, 2010; Kalapotharakos et al., 2012).

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One limitation of force-free, MHD, and even the resistive simulations is that they do not address the creation and acceleration of particles in the magnetosphere. These particles are created via pair-production and accelerated in vacuum gaps (Sturrock, 1971; Arons and Scharlemann, 1979; Cheng et al., 1986; Muslimov and Harding, 2003), where the force-free assumption breaks down ($\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$). To model the effect of pair production on the global structure of the magnetosphere and gain a deeper understanding of magnetospheric emission, it is natural to perform particle-based simulations.

A well-developed technique for electromagnetic simulation using particles is the Particle in Cell (PIC) method e.g. Birdsall and Langdon (1991). The major reason for using PIC is that it offers a kinetic and self-consistent approach to the solution of Maxwell's equations in a plasma. The PIC method has already been used to simulate physics relevant to pulsar magnetospheres including pair production and particle acceleration in vacuum gaps (Timokhin, 2010; Timokhin and Arons, 2013) and instabilities in electron–positron current sheets (Zenitani and Hoshino, 2007, 2008). These simulations were local, however, and it is our aim to model the pulsar magnetosphere using global PIC simulations.

Philippov and Spitkovsky (2013) have simulated the magnetosphere of the aligned rotator using a relativistic, electromagnetic 3D Cartesian PIC code and found that their results are consistent with force-free simulations to $\sim 10\%$. However, Cartesian simulations of pulsar magnetospheres are prohibitively expensive to perform in terms of computational cost. The fundamental reason for this is that Cartesian coordinates are far from ideal for simulating a physical system, which is most naturally described in spherical geometry. Wada and Shibata (2011) proposed a purely particle-based approach (not PIC) that did away with the computational mesh altogether. However, their method is electrostatic at its core, which limits its applicability. Chen and Beloborodov (2014) have also performed 2.5D axisymmetric PIC simulations of the pulsar magnetosphere. They found that pair production all the way out to the light cylinder was necessary to generate a force-free magnetosphere, showing that global PIC modeling of the pulsar magnetosphere can lead to fundamental insights and help test theoretical models.

Our aim in this paper is to demonstrate the efficiency and accuracy of PIC simulations for modeling the pulsar magnetosphere. In this paper, we do not simulate the aligned dipole rotator, but rather present a suite of test problems that is useful for validating a PIC code used to simulate the pulsar magnetosphere. We use this suite to validate our new code, *PICsar*. In addition to validating *PICsar*, we also compare and contrast the applicability of various standard PIC algorithms with respect to the pulsar problem. In some cases, we find that the choice of algorithm can have *drastic* consequences for the solution. Understanding the differences between various algorithms and being aware of which are the good ones to use is essential for successfully simulating the pulsar magnetosphere using PIC.

Our new code, *PICsar*, is a 2.5D axisymmetric, relativistic, electromagnetic, charge conservative PIC code that is several orders of magnitude faster for axisymmetric pulsar simulations than a 3D Cartesian PIC code. *PICsar* implements coordinates that are body-fitted to the surface of a sphere. This makes the boundary condition on the surface of the star simple to implement and eliminates the need for simulating the plasma in the neutron star interior. Furthermore, *PICsar* implements a radiation absorbing outer boundary condition, which allows electromagnetic waves to leave the simulation domain and set up a steady state dynamically.

This paper is organized as follows. In Section 2 we present an overview of *PICsar*, and in Section 3 we give a detailed account of the algorithms used in *PICsar*. In Section 4 we present tests of *PICsar*, which demonstrate the accuracy of the code and its capabilities. In Section 5 we present simulations of the aligned monopole rotator and present an algorithm for charge injection. Finally, we discuss our results in Section 6.

2. *PICsar* overview

PICsar is a 2.5D axisymmetric, relativistic, electromagnetic, charge conservative PIC code with a radiation absorbing outer boundary. It is based on the 3D Cartesian PIC code TRISTAN (Buneman, 1993), but has been heavily modified to work in 2.5D axisymmetry and has been parallelized using MPI.

A PIC code is a type of particle-mesh code in which electric and magnetic fields are stored and updated on a grid using Maxwell's equations (Birdsall and Langdon, 1991). The moniker "2.5D axisymmetric" means that there are in general six nonzero field components (three of electric and three of magnetic field) at each point, but that the azimuthal derivative, $\partial/\partial\phi$, of any field quantity is equal to zero. From a computational point of view, this means that a 2D grid $(r-\theta)$ of 3D vectors needs to be simulated, which explains the use of the term 2.5D.

In an electromagnetic PIC code, the equations solved to advance the fields from one time step to the next are the time-dependent Maxwell's equations. These are written most compactly in Lorentz–Heaviside units, and we will use Lorentz–Heaviside units throughout, unless explicitly stated. The time-dependent Maxwell's equations in Lorentz–Heaviside units are

$$\begin{aligned}\frac{\partial \boldsymbol{B}}{\partial t} &= -c\nabla \times \boldsymbol{E} \\ \frac{\partial \boldsymbol{E}}{\partial t} &= c\nabla \times \boldsymbol{B} - \boldsymbol{J}\end{aligned} \tag{1}$$

and the time-independent Maxwell equations are

$$\begin{aligned}\nabla \cdot \boldsymbol{E} &= \rho \\ \nabla \cdot \boldsymbol{B} &= 0.\end{aligned} \tag{2}$$

Only the time-dependent Maxwell equations are solved by the code, and the time-independent equations form a pair of constraints. The Yee algorithm (Yee, 1966), which is second order in both space and time is used to update the E and B fields and ensures that if the constraint Eq. (2) are satisfied initially, then they are satisfied for all time (to machine precision) in the absence of sources (i.e. charged particles).

In order to satisfy Eq. (2) in the presence of sources, the current deposited to the numerical grid must satisfy the equation of charge conservation. If the only sources are particles and the nth particle has a shape function (i.e. spatial charge distribution) given by $\rho_n(\boldsymbol{x} - \boldsymbol{x}_n)$, where $\boldsymbol{x}_n$ is the particle position, then the equation of charge conservation is

$$\sum_n \frac{\partial \rho_n}{\partial t} = -\nabla \cdot \boldsymbol{J}. \tag{3}$$

For charge conservative current deposition in *PICsar*, we use the Villasenor–Buneman algorithm (Villasenor and Buneman, 1992), which takes the particles to have the same shape function as the grid cells. Thus, in a curvilinear coordinate system the shape function of the particles is not constant, but changes to reflect the local shape and stretch of the coordinate system.

From Eq. (3), it is clear that as the particles move around the grid, they deposit current. The job of the particle mover in a PIC code is to move and accelerate the particles from one timestep to the next. In a relativistic, electromagnetic PIC code, particle motion is dictated by the Lorentz force law

$$\gamma m \frac{d\boldsymbol{v}}{dt} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right). \tag{4}$$