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A stochastic approach to Galactic proton propagation: Influence of the spiral arm structure

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HIGHLIGHTS

- SDE code for multidimensional Fokker–Planck-type equations.
- Applied to Galactic propagation of energetic protons.
- Diffusion coefficient reflects spiral arm structure.
- New diffusion coefficient leads to non-negligible effects on the spectrum.
- And to enhance flux ratios between inarm and interarm regions.

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ABSTRACT

A newly developed numerical code solving Fokker–Planck type transport equations in currently four dimensions (space plus momentum or energy) and time by means of stochastic differential equations (SDEs) is applied to the Galactic propagation of Cosmic Ray protons, where the paths of pseudo particles originating in spiral arms and in the interarm region are traced back to a distribution of point sources. The transport equation in this first, simplified approach includes scalar spatial diffusion and catastrophic energy losses. In a first step we validate the code by obtaining results being consistent with previous ones obtained with finite-difference methods, revealing lower spectra with less variations in the interarm region compared to the inarm region. While Effenberger et al. (2012) used this approach to study the effects of a fully anisotropic diffusion tensor, we concentrate here on a diffusion coefficient taking into account the spiral arm structure. Such a variable diffusion coefficient is of importance e.g. for the long-scale time-variation of the Cosmic Ray flux at Earth related to spiral arm crossings. For our choice of parameters, we find that a diffusion coefficient reflecting the spiral arm structure leads to a clearly enhanced flux ratio between the inarm and interarm regions. The strength of this effect depends, however, on the parameters chosen.

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1. Introduction

An important boundary condition for modelling the transport of Cosmic Rays (CRs) through the heliosphere (e.g. Strauss et al., 2011, and references therein) is the local interstellar spectrum (LIS). Since it still cannot be measured (cf. discussion in Herbst et al., 2012), Galactic propagation models like the GALPROP code (Strong et al., 2007) have to be applied. As pointed out, however, by Bisschoff et al. (2011) the finite-differences approach of the latter encounters some serious numerical restrictions when going to more than two dimensions plus time. A different numerical approach was presented by Zhang (1999), who applied the method of stochastic differential equations (SDEs) to heliospheric CR transport.

Recently, we developed a general numerical code, based on the SDE approach (cf. Kopp et al., 2012, also for a description of the method), that solves general Fokker–Planck type transport equations in currently four dimensions (space plus momentum or energy) plus time. As shown by Kopp et al. (2012) for a test case and by Effenberger et al. (2012) for Galactic propagation, the code is able to address and deploy a full 3D diffusion tensor, which is not yet possible in the GALPROP code. In this paper, however, we first validate our new code with the results by Büsching and Potgieter

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(2008) and, thus, restrict ourselves to scalar diffusion, but take into account the time-dependent distribution of point sources used in the latter approach.

In order to go a step further towards a full Galactic propagation code, we allow the diffusion coefficient to be a function of the density structure of the spiral arms of the Galaxy. The spiral arm structure might be responsible for variations of the CR flux at Earth on very long time scales. Shaviv (2003) reported a flux ratio between the Earth being located in the inarm and the interarm regions can reach a peak value of 300%. In contrast, Sloan and Wolfendale (2013) estimate this ratio assuming a constant diffusion coefficient, i.e. independent of the spiral arm structure, and obtain that this ratio cannot exceed 30%.

The paper is organised in the following way: after a description of the basic numerical method we describe our Galactic propagation model. The next section compares the results obtained with the new numerical approach with those by Büsching and Potgieter (2008) for a spatially constant diffusion coefficient and demonstrate the influence of a spatially variable diffusion coefficient in particular with regard to the findings by Sloan and Wolfendale (2013), before we summarise our results.

2. The numerical approach

Transport equations for the differential intensity $N(\vec{r},p,t)$ of CRs like the Parker equation (Parker, 1965) for the heliosphere are in general Fokker–Planck type equations of the form

$$\frac{\partial N}{\partial t} = \vec{\nabla}\cdot\left(\mathcal{K}\cdot\vec{\nabla}N - \vec{V}N\right) - \vec{W}\cdot\vec{\nabla}N + \Omega\frac{\partial N}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(D_{pp}\left(\frac{\partial N}{\partial p} - \frac{2}{p}N\right)\right) - LN + S. \quad (1)$$

N is a function of spatial position $\vec{r}$, (isotropically distributed) momentum p, and time t and is related to the phase space density f via $N(\vec{r},p,t) = p^2 f(\vec{r},p,t)$. Here, $\mathcal{K}$ stands for the spatial diffusion tensor, D_{pp} for the momentum diffusion coefficient, Ω is the (e.g. adiabatic) momentum loss term, and the velocity has been split into a conservative ($\vec{V}$) and a non-conservative ($\vec{W}$) part. The two last terms describe linear (e.g. catastrophic) losses L and source term S.

The SDE approach considers the propagation of phase space elements, which are also referred to as pseudo particles, where "pseudo" indicates that they follow the mathematical description of the physical situation rather than the physical situation itself. The propagation equations read

$$d\vec{r} = \vec{A}dt + \mathcal{B}\cdot d\vec{W}$$
$$dp = A_p dt + B_{pp} dW_p, \quad (2)$$

where $\vec{A}$ and A_p describe the convective part of the propagation in physical and momentum space, respectively. $d\vec{W}$ (and dW_p accordingly) is of the form

$$d\vec{W} = \vec{h}\sqrt{dt}, \quad (3)$$

with dt being the time-step and and $\vec{h}$ a vector of Gaussian distributed random numbers in the range $(-\infty,\infty)$. Due to the finite number representation on computers this range, however, does not go beyond the order of ten (cf. Kopp et al., 2012, for details). The tensor $\mathcal{B}$ is the (non-unique) square root of the spatial diffusion tensor, which has to obey the relation

$$\mathcal{B}\,{}^t\mathcal{B} = \mathcal{K} + {}^t\mathcal{K} \quad (4)$$

with ${}^t\mathcal{X}$ denoting the transpose of tensor $\mathcal{X}$. Accordingly, $B_{pp} = \sqrt{2D_{pp}}$. The momentum diffusion was not taken into account in our present Galactic application and is, thus, omitted in the following.

In contrast to traditional finite-differences methods, the SDE can be solved forward as well backward in time. Note that the same SDE applied forward in time solves a different transport as when applied backward in time, so that the forward and backward SDEs differ if referring to the same transport equation:

- Time-forward equation:

$$\frac{\partial N}{\partial t} = \vec{\nabla}\cdot\left(\vec{\nabla}\cdot({}^t\mathcal{K}N)\right) - \vec{\nabla}\cdot\left(\vec{A}_F N\right) - \frac{\partial}{\partial p}(-\Omega N) - L_F N + S \quad (5)$$

- Time-backward equation:

$$\frac{\partial N}{\partial t} = \mathcal{K} : \left(\vec{\nabla}\otimes\vec{\nabla}N\right) + \vec{A}_B\cdot\vec{\nabla}N + \Omega\frac{\partial N}{\partial p} - L_B N + S, \quad (6)$$

where the symbols : and $\otimes$ denote the scalar product of two tensors and the tensor product of two vectors, respectively. Due to the fact that the second order derivatives can always be made symmetric (Schwartz's theorem), Eq. (4) is the same for both directions of integration, so that an asymmetry in $\mathcal{K}$ is visible only in the convective terms. The abbreviations used in these equations for the convective and linear loss terms[2] are:

$$\begin{aligned}
\vec{A}_F &= \vec{\nabla}\cdot{}^t\mathcal{K} + (\vec{V}+\vec{W})\\
\vec{A}_B &= \vec{\nabla}\cdot\mathcal{K} - (\vec{V}+\vec{W})\\
L_F &= L - \vec{\nabla}\cdot\vec{W} + \frac{\partial\Omega}{\partial p}\\
L_B &= L + \vec{\nabla}\cdot\vec{V}
\end{aligned} \quad (7)$$

In this work we consider particles at a given time $t = t_0$ being located within spiral arms or in the interarm regions and trace them back in time until $t = 0$ is reached. The linear loss terms L (L_B for the time-backward integration applied here and L_F for the time-forward integration) as well as the sources S are taken into account by means of a path amplitude a and a path weight α (cf. Zhang, 1999; Kopp et al., 2012): the amplitude is initialised with $a_0 = 0$, the weight with $\alpha_0 = 1$. If the pseudo particle at time $t_j = t_0 - jdt$ "sees" a loss term L_j, the weight at this time, α_j, is reduced (or enhanced if L_j is negative) according to the relation ($j-1$ stands for the previous time-step):

$$\alpha_j = \alpha_{j-1}\ \exp(-L_j dt) = \exp\left(-\sum_{k=0}^{j} L_k dt\right). \quad (8)$$

The particle amplitude a_j uses this path weight to take into account source encounters:

$$a_j = a_{j-1} + S_j\alpha_j = a_{j-1} + S_j\exp\left(-\sum_{k=0}^{j}L_k dt\right) = \sum_{l=0}^{j}\left\{S_l\exp\left(-\sum_{k=0}^{l}L_k dt\right)\right\} \quad (9)$$

where S_j stands for the source term (cf. Eq. (13) below) at time $t = t_j$.

3. The Galactic propagation model

If applied to the Galactic propagation of protons and assuming scalar diffusion, transport equation (1) reduces to

[2] Note the typo in Kopp et al. (2012) in the linear loss terms.

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