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Time variability of viscosity parameter in differentially rotating discs

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HIGHLIGHTS

- Fluctuation in viscosity parameter as a nonlinear perturbation.
- Reduction of nonlinear perturbation equations to a dynamical system.
- Identification of growing modes of viscosity fluctuation with saturation and final degradation.
- Analysis of the viscosity fluctuation mechanism in physically acceptable range of mean-flow parameters.
- Identification of the growing phase of viscosity fluctuation as a power law in time.

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ABSTRACT

We propose a mechanism to produce fluctuations in the viscosity parameter (α) in differentially rotating discs. We carried out a nonlinear analysis of a general accretion flow, where any perturbation on the background α was treated as a passive/slave variable in the sense of dynamical system theory. We demonstrate a complete physical picture of growth, saturation and final degradation of the perturbation as a result of the nonlinear nature of coupled system of equations. The strong dependence of this fluctuation on the radial location in the accretion disc and the base angular momentum distribution is demonstrated. The growth of fluctuations is shown to have a time scale comparable to the radial drift time and hence the physical significance is discussed. The fluctuation is found to be a power law in time in the growing phase and we briefly discuss its statistical significance.

1. Introduction

The presence of accretion discs around compact objects like the neutron star and the black hole in both galactic and extragalactic X-ray sources is now a well established phenomenon. Some radio objects such as active galactic nuclei have accretion discs around supermassive black holes. Apart from several details such as environment, size, strength of magnetic field and cooling mechanism, all the global models of accretion systems share a common hydrodynamic structure. The central idea being, the turbulent shear stress causes dissipation of angular momentum and energy of the rotating fluid particles such that accretion can take place. The origin of turbulence and hence turbulent viscosity was an issue for the founders of the field of 'accretion powered astrophysical systems' and still remains to be a major issue. The closure model proposed by Shakura and Sunyaev (1973) remains the only working model for turbulent shear stress in astrophysical accretion discs. In this model, the physics behind the turbulent shear stress is parameterised by a dimensionless number α. Thus the α viscosity continues to be the central idea in any model for hydrodynamic transport in accretion systems.

The spirit of the α viscosity is as follows: any eddy velocity which is greater than the local sound speed will dissipate quickly and cannot be the cause of eddy viscosity. Hence the turbulent stress must be less than the local isotropic pressure. Thus the shearing stress is taken to be proportional to the local isotropic pressure where the proportionality factor is called α, where $0 < \alpha < 1$. When $\alpha \sim 1$ the flow is called a high viscosity flow, whereas when $\alpha \lesssim 10^{-2}$, the flow is called a low viscosity flow. With this model, the spectrum of cool Keplerian discs could be explained (Pringle and Rees, 1972; Novikov and Thorne, 1973; Shakura and Sunyaev, 1973). The idea of a sub-Keplerian disc was proposed to explain the nonthermal tail of the spectra from X-ray sources (Liang and Thompson, 1980; Paczyńsky and Bisnovatyi-Kogan, 1981; Muchotrzeb and Paczyńsky, 1982). In

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the case of a sub-Keplerian accretion disc the turbulent energy dissipated locally, is partly advected radially and partly emitted as radiation via nonthermal processes. In this model also, α closure remained unchanged although additional ram pressure was added to the total pressure (Chakrabarti and Titarchuk, 1995; Mandal and Chakrabarti, 2005; Rajesh and Mukhopadhyay, 2010). Since α is the ratio of two flow variables, namely, the turbulent shear stress and isotropic pressure, α should also be considered as a flow (continuum) variable. Since there is no known equation for the evolution of α, it is treated as a disc parameter, and its value is fixed globally.

Apart from the fundamental problem to explain the origin of turbulent viscosity, there are other phenomena which await complete understanding, such as rapid X-ray variabilities in black hole accretion discs, aperiodic X-ray fluctuations and quasi periodic oscillations (QPOs) in accretions discs. Global mode oscillations and waves are invoked to explain some of these phenomena (Mukhopadhyay, 2009). As the α viscosity is the source of energy dissipation in accretion discs, it is logical to attribute some of the time variabilities of the spectra to the temporal variation of α. For example, in order to explain the $1/f$ (flicker) noise in X-ray sources, Lyubaraskii (1997) considered the local temporal fluctuations of α at outer radii. This would cause a change in mass accretion rate at inner radii where most of the X-rays are emitted. In order to have such an effect, a time varying component of α was assumed. The fluctuation was assumed to grow enough in a local accretion time scale. Thus the fluctuations in α resulted in a variable mass accretion rate, which would lead to variations in X-ray luminosity. Although the ultimate aim of our work in the present paper is similar to that of Lyubaraskii (1997), i.e., to study a mechanism to produce variabilities in the observed luminosities (say, X-ray) from accretion discs, our approach is somewhat different, and may be stated as follows: considering a steady state accretion in an annular region of an accretion disc with self-similar base flow profiles, we wish to study how the background α changes in response to any perturbation on the radial velocity field? Such perturbations of the radial velocity field (i.e., the mass accretion rate) may, in general, be of internal or external origin. These kind of studies have direct implications on the observed variabilities in X-ray luminosities from accretions discs.

A stable accretion system tries to maintain the steady mass flow across all radii. Any cause, internal or external which disturbs this steady state, will be quickly nullified by viscous dissipation. In Section 2.1 we model the steady state flow variables in a local annular region as a power law in radial coordinate. The global flow domain can be thought of as a collection of such annular regions. In Section 2.2 the evolution equations for perturbations in the mean density and the radial velocity, causing perturbation in α are discussed. We use the standard α model for viscous stress. In Section 2.3 we reduce the perturbation equations to a set of nonlinear dynamical systems of equations, by specialising to the case when the Lagrangian derivatives (defined with respect to the radial velocity field) of the perturbations in the flow variables vanish. In Section 3 we demonstrate that the growth of the viscosity parameter is always followed by saturation and degradation, and the fluctuation asymptotically goes to zero. The behaviour of the fluctuation is strongly dependent on the radial location, base angular momentum distribution and the mean viscosity of the flow. We demonstrate that the growth of the fluctuation in viscosity parameter always scales as the local accretion time, and that it shows a power law growth phase in time, in the astrophysically relevant time scale. We conclude in Section 4.

2. Model equation describing the system and the solution procedure

Let us consider a cylindrical coordinate system, with spacetime coordinates denoted by (r, ϕ, z, t), whose origin is at the centre of the compact object. The angular velocity vector, Ω, is pointed along z (vertical) direction, and the midplane of the accretion disc is at $z = 0$. We begin by considering a vertically integrated, axisymmetric, steady-state accretion flow, in which, we focus on an annular region of the accretion disc. We consider axisymmetric perturbations on base radial velocity field and mean density in this annular region to study the response of such perturbations on the evolution of the viscosity parameter. Thus all the base flow variables in this study are functions only of the radial coordinate (r), whereas the perturbations depend on both, the radial coordinate (r) and time (t).

2.1. Base flow

For a general accretion flow, we consider a small annular region specified by the mean velocity field, where V and Ω are the magnitudes of the radial and angular velocity fields, respectively. Let us specify the unperturbed axisymmetric, steady-state accretion (base) flow where the radial velocity and the angular velocity are power laws in radial coordinate, i.e., $V = V_0 r^{-j}$ and $\Omega = \Omega_0 r^{-q}$. The explicit radial dependence of other fluid variables in an unperturbed state can be obtained by solving the conservation equations for mass, radial momentum and angular momentum, given as:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\Sigma V) = 0 \tag{1}$$

$$\Sigma\left(V\frac{\partial V}{\partial r} - r\Omega^2\right) = \frac{k\Sigma}{r^2} - \frac{\partial P}{\partial r} \tag{2}$$

$$\frac{\Sigma V}{r^2}\frac{\partial}{\partial r}(r^2\Omega) = -\frac{1}{r^3}\frac{\partial}{\partial r}(r^3 W^{\phi r}) \tag{3}$$

where Σ and P are vertically integrated density and pressure, respectively. The quantity $k = -GM$, where G is the universal gravitational constant and M is the mass of the central object. We solve the above set of equations along with the equation of state, $P = \Sigma T$, where T is the effective temperature of the flow. We impose the boundary condition that all the physical quantities go to zero as $r \to \infty$. For the turbulent stress, we use the α viscosity model, i.e., $W^{\phi r} = \alpha P/r$, where α is the Shakura–Sunyaev viscosity parameter. We can write the solution to the above set of equations as,

$$V(r) = V_0 r^{-j}; \quad \Omega(r) = \Omega_0 r^{-q}; \quad \Sigma(r) = \Sigma_0 r^{j-1} \tag{4}$$

$$T(r) = \left(\frac{k}{j-2}\right)r^{-1} - \left(\frac{jV_0^2}{j+1}\right)r^{-2j} + \left(\frac{\Omega_0^2}{j-2q+1}\right)r^{2(1-q)} \tag{5}$$

where $V_0 < 0$, as the radial flow is directed towards the central object, and $\Sigma_0 = \dot{M}/(4\pi V_0)$, where $\dot{M}$ is a negative quantity called the mass accretion rate. Since both, the radial velocity and density, decrease with increasing values of r, we get from Eq. (4) that $0 < j < 1$. The value of q indicates the angular momentum distribution of the base flow; $q = 1,\ q = 3/2$ and $q = 2$ describe, respectively, the flat rotation, Keplerian rotation and constant angular momentum disc profiles. From the angular momentum balance equation, we get

$$\alpha(r)T(r) = -V_0\Omega_0 r^{1-q-j} + \frac{c}{\Sigma_0}r^{-j-1} \tag{6}$$

As αT is a physical quantity, it approaches zero as $r \to \infty$, according to the boundary condition that we have chosen. In Eq. (6), since the term containing the integration constant c goes to zero as $r \to \infty,\ c$ is nonzero in general. The physics behind c comes from the actual physical mechanism which produces the α viscosity. The origin of α is beyond the scope of the present analysis, therefore, we can only choose α at a particular radius which automatically fixes the value of c from Eq. (6).