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Why the Big Bang Model does not allow inflationary and cyclic cosmologies though mathematically one can obtain any model with favourable assumptions

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ABSTRACT

Various versions of standard Big Bang Model (BBM) including the current LCDM cosmology require an "inflationary" phase for the nascent universe ($\Delta t \sim 10^{-32}$ s) during which the size of the universe blows up by a factor of $\sim 10^{78}$. However, the so-called $R_h = ct$ cosmology (Melia, 2013a) claims that the isotropy and homogeneity of the present universe can be understood without assuming any inflationary phase. To this effect, Melia and his coworkers have often invoked "Weyl's Postulate" and "Birkhoff's Theorem" to qualitatively argue for this novel model. On the other hand, here, we explore for a cogent analytical basis of the $R_h = ct$ proposal which is claimed to have such a profound implication. First we show that (i) if the spatial flatness of the BBM would be presumed, $R_h = ct$ cosmology may indeed follow. To further explore this issue without prior assumption of flatness (ii) we equate the twin expressions for the Energy Complex (EC) associated with BBM computed by using the same Einstein pseudo-tensor and quasi-Cartesian coordinates (Mitra, 2013b). This exercise surprisingly shows that BBM has *tacit and latent* self-consistency constraints: it is spatially flat and its scale factor $a(t) \propto t$. Accordingly, it seems that, there is no scope for the other models including inflationary and cyclic ones. The real lumpy universe may be too complex for the simplistic Big Bang model.

1. Introduction

Modern cosmology is based on the hypothesis that the observed universe is isotropic and homogeneous to all fundamental (comoving) observers on all scales. Further, observations apparently suggest that it is expanding. The isotropic form of the space time geometry of such an universe is given by the Friedmann Robertson Walker (FRW) metric:

$$ds^2 = dt^2 - \frac{a^2(t)}{f^2}[dr^2 + r^2 d\Omega^2] \tag{1}$$

where $a(t)$ is the scale factor of the universe, t is the universal cosmic time, r is a comoving radial coordinate, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and

$$f = 1 + kr^2/4 \tag{2}$$

Here the normalized spatial curvature parameter k can assume values of $0, +1$, or -1. Although the FRW metric has traditionally been derived by using geometrical symmetry arguments, it is interesting to note that it can be derived by directly solving Einstein equations for a spherically evolving uniform density perfect fluid (Mitra, 2012). Following the Big Bang, the universe is supposed to expand with a deceleration due to self-gravity. In such a case, one would tentatively expect a scale factor

$$a(t) \sim t^n \quad \text{where } n < 1 \tag{3}$$

Despite such an assumed isotropy and homogeneity, it is expected that immediately after the "Big Bang", the cosmic plasma would be highly turbulent and heterogeneous. However, the Cosmic Microwave Background (CMB) appears to be extremely homogeneous and isotropic. In particular even two patches of CMB on the diametrically opposite sides of the sky have exactly the same temperature and pattern. This is unexpected in view of the initial turbulence and anisotropy. In particular the size of the particle horizon, the region of causal influence, could be as low as 62 cm (Narlikar, 2010). This puzzle is known as the "Horizon Problem".

If the universe is intrinsically flat ($k = 0$), its density always is pegged to its critical value $\rho(t) = \rho_c(t)$. But if $k \neq 0$, then $\rho(t) \neq \rho_c(t)$. Observations show that, in the present epoch $\rho(t_0) \sim \rho_c(t_0)$ definitely within a factor of few. However such an

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equality within an order of magnitude requires that in the past grand unification time (t_*) having a temperature $\sim 10^{15}$ GeV, the flatness of the universe $k/a^2(t_*) \to 0$ as $\rho(t_*)$ must be fine tuned to $\rho_c(t_*)$ by a factor of $\sim 10^{-53}$:

$$\Omega_* - 1 \sim 10^{-53}(\Omega_0 - 1) \quad \Omega = \rho/\rho_c \tag{4}$$

And this puzzle is known as "Flatness Problem". To resolve such puzzles, it was hypothesized that, immediately after the Big Bang, may be after $t_* \sim 10^{-35}$ s, the universe went through a phase of extremely rapid exponential expansion (Narlikar, 2010; Guth, 1981) for a duration of around $\Delta t \sim 10^{-32}$ s during which the scale factor increased by a factor of $\sim 10^{26-28}$. Such a remarkable phenomenon is known as cosmic "inflation". There are innumerable research papers, review articles and books on "inflation" and here we just refer to two books (Liddle and Lyth, 2000; Mukhanov, 2005). With an exponentially expanding space, two nearby patches in the nascent turbulent universe get separated at *extreme superluminal speed*, so much so that the distance between them quickly exceeds the limits of communications. This solves the horizon problem. On the other hand the sudden increase of initial scale factor by a factor of $\sim 10^{26}$ reduces the expected spatial curvature of the present universe by a factor of $\sim 10^{52}$. And this resolves the flatness problem.

Inflation is supposed to be driven by a suitable "inflaton field" which has a vacuum like equation of state $p = -\rho$ $(G = c = 1)$. Though inflation implies a phase of a hyper accelerated expansion, universe is assumed to revert to its normal decelerated expansion soon thereafter. And one needs to invent a tailor made "hill-top" potential or "waterfall transition" for the inflaton field to explain a "graceful exit" from hyper acceleration to normal deceleration phase. Again there are many models for such "graceful exit" (Liddle and Lyth, 2000; Mukhanov, 2005). In recent times, many of such models are inspired by string theory such as Cicoli and Mazumdar (2011). In fact recent Planck results are claimed to have reconfirmed such an inflationary history in the early cosmic history (Ade et al., 2013). However, there are no unanimity about the fundamental aspects of inflation such as the right initial conditions necessary to trigger it, and the precise physics which caused the super prompt "graceful exit". In fact one of the fathers of this paradigm now refutes the inflation hypothesis (Steinhardt, 2011). Similarly Penrose too does not believe in this paradigm (Penrose, 1989). It is even claimed that the Planck data is actually in tension with the inflation hypothesis (Ijjas et al., 2013) in contradiction to the claim made in Ade et al. (2013). The objective of this paper is not to take side with these latter authors, but, on the other hand, the objective is note the fact there is no clarity or unanimity about the inflation hypothesis.

Without the assumption of inflation, it is believed that horizon and flatness problems would cripple the present version of BBM, namely the *LCDM* model too by which, in the recent epoch, universe is again in a mode of accelerated expansion. This model was motivated by the observation that Type 1a supernovae appear to be fainter than what would be expected in a decelerating universe (Perlmutter et al., 1999; Riess et al., 1998), and the acceleration is supposed to be caused by an unknown repulsive "dark energy". It is claimed that as per latest Planck data dark energy makes up 68.3% of the energy density of the Universe, a slightly smaller proportion than WMAP had estimated. On the other hand the contribution of (cold) dark matter (CDM) is now estimated as 26.8%, leaving normal matter making up less than 5% (Ade et al., 2013).

Interestingly it has recently been claimed that the need for a brief yet severe inflationary phase arises in order to compensate for the deceleration present in the standard BBM. On the other hand, it is also claimed that a BBM free from such a secular deceleration, such as the $R_h = ct$ cosmology, can explain the isotropy and homogeneity of the observed universe without invoking any inflation (Melia, 2013a). The reader is directed to some previous publications for gauging the foundations of the $R_h = ct$ cosmology (Melia and Shevchuk, 2012; Melia, 2012a). And the aim of the present paper is not to dissect this paper (Melia, 2013a); on the other hand, the aim is to explore the very foundations of $R_h = ct$ cosmology in a rigorous manner.

1.1. Ideas behind $R_h = ct$ cosmology

By Newtonian gravity Gauss's theorem for a spherically symmetric mass distribution, a test mass at an interior radius $R = R$ is affected only by the interior mass $M(R)$ and remains unaffected by the exterior mass distribution. As per Melia, a similar thing should happen in general relativity too (Melia, 2013a, 2012a; Melia and Shevchuk, 2012). Further, by Birkhoff's theorem, the exterior space time of an adiabatically evolving spherical fluid is given by the static Schwarzschild solution

$$ds^2 = (1 - 2M_b/R)dT^2 - (1 - 2M_b/R)^{-1}dR^2 - R^2 d\Omega^2 \tag{5}$$

Inspired by such facts, Melia claims that there is a "Corollary of Birkhoff's Theorem" for an isotropic universe, by which any interior cosmic observer should feel only *gravity of the interior region rather than gravity by the exterior mass distribution*. And for any given observer, there should be a "Cosmological Horizon" like the Schwarzschild black hole event horizon, at $R = 2M$ where $g_{TT} = 0$ and $-g_{RR} = \infty$. Though Melia's intuition may be correct, the application of Birkhoff's Theorem is not appropriate here because, no section of the universe has an external vacuum with zero matter energy momentum ($T^a_b = 0$) nor is there any asymptotic flat spacetime for the universe.

Next Melia tries to justify this notion of a "Cosmic Horizon" by citing the vacuum de-Sitter metric expressed in Schwarzschild coordinates (Melia and Shevchuk, 2012; Melia, 2012a):

$$ds^2 = (1 - 2M/R)dT^2 - (1 - 2M/R)^{-1}dR^2 - R^2 d\Omega^2 \tag{6}$$

where $M = \Lambda R^3/6$. But again, once one assumes a FRW universe with $\rho = \rho(t)$, the example of the de-Sitter spacetime with $\rho_{vac} = \Lambda/8\pi$ is not appropriate. In any case, Melia postulates a gravitational horizon for a certain observer defined by (after recalling G and c):

$$R_h = \frac{2GM_h}{c^2} = \frac{8\pi G}{3c^2}\rho(t)R_h^3 \tag{7}$$

Then Melia notes that with a value of Hubble's parameter $H \approx 70$ km/s/Mpc, one obtains $R_h \approx 13.3$ billion light year (assuming $k = 0$) which is close to ct_0, the distance travelled by Big Bang photons till now because the estimated age of the universe $t_0 \approx 13.3$ billion year. He feels that this is not a mere coincidence, and one must intrinsically have $R_h = ct$ for all epochs. Finally he justifies this conclusion by claiming that any other relationship between R_h and ct would violate the "Weyl's Postulate" (Melia, 2013a, 2012a; Melia and Shevchuk, 2012).

Clearly there are loose ends for these strings of arguments which led to the postulation of this unique form of BBM. Accordingly several authors have questioned the foundations of $R_h = ct$ cosmology from various angles. For instance Lewis and Oirschot claimed that the concept of a "Cosmic Horizon" is flawed for LCDM model having a phantom dark energy ($p < -\rho$) (Lewis and van Oirschot, 2012). But Melia refuted this claim (Melia, 2012b). Next Bilicki and Seikel questioned "the very foundations of the model and its consequences for the evolution of the Universe" (Melia, 2012b). They also claimed that "the discussed model is strongly disfavoured by observations, especially at low redshifts ($z < 0.5$)" (Bilicki and Seikel, 2012). The latter part has however been force-