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On the gravitational redshift

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HIGHLIGHTS

- Gravitational redshift is still an important subject in modern physics.
- There is no consensus on the physical process(es) causing the shift.
- Solution is formulated in analogy with Fermi's treatment of the Doppler effect.
- Physical processes with conservation of energy and momentum result in observed shift.
- Gravitational field affects the release of photon and not atomic transition.

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ABSTRACT

The study of the gravitational redshift—a relative wavelength increase of $\approx 2 \times 10^{-6}$ was predicted for solar radiation by Einstein in 1908—is still an important subject in modern physics. In a dispute whether or not atom interferometry experiments can be employed for gravitational redshift measurements, two research teams have recently disagreed on the physical cause of the shift. Regardless of any discussion on the interferometer aspect—we find that both groups of authors miss the important point that the ratio of gravitational to the electrostatic forces is generally very small. For instance, the ratio of the gravitational force acting on an electron in a hydrogen atom situated in the Sun's photosphere to the electrostatic force between the proton and the electron in such an atom is approximately 3×10^{-21}. A comparison of this ratio with the predicted and observed solar redshift indicates a discrepancy of many orders of magnitude. With Einstein's early assumption that the frequencies of spectral lines depend only on the generating ions themselves as starting point, we show that a solution can be formulated based on a two-step process in analogy with Fermi's treatment of the Doppler effect. It provides a sequence of physical processes in line with the conservation of energy and momentum resulting in the observed shift and does not employ a geometric description. The gravitational field affects the release of the photon and not the atomic transition. The control parameter is the speed of light. The atomic emission is then contrasted with the gravitational redshift of matter–antimatter annihilation events.

1. Introduction

The study of the gravitational redshift, a relative wavelength increase of $\Delta\lambda/\lambda \approx 2 \times 10^{-6}$ was predicted for solar radiation by Einstein (1907), is still an important subject in modern physics (Kollatschny, 2004; Lämmerzahl, 2009; Chou et al., 2010; Turyshev, 2013). Jewell (1896) had found in electric arc spectra:

> "[...] that the metallic lines were almost invariably displaced toward the violet, when compared with the corresponding solar lines."

At that time—in 1896—a high pressure in the solar atmosphere was erroneously considered as causing the shift (c.f., LoPresto et al., 1991). Measurements of the gravitational redshift of solar spectral lines are inherently difficult, because high speeds of the emitting plasmas in the atmosphere of the Sun lead to line shifts due to the classical Doppler effect. Improved observational techniques (cf., e.g. LoPresto et al., 1980; Cacciani et al., 2006; Takeda and Ueno, 2012), have nevertheless established a shift of

$$c_0 \frac{\Delta\lambda}{\lambda} \approx 600 \text{ m s}^{-1}, \quad (1)$$

where $c_0 = 299\ 792\ 458 \text{ m s}^{-1}$ is the speed of light in the vacuum (Bureau International des Poids et Mesures, 2006) remote from any masses. This shift is consistent with Einstein's General Theory

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of Relativity (GTR) (Einstein, 1916). Together with various other aspects of GTR—from the deflection of light by a gravitational centre (Einstein, 1911, 1916; Dyson et al., 1920; Mikhailov, 1959; Shapiro et al., 2004) to Mercury's perihelion precession (Le Verrier, 1859; Einstein, 1915; Nobili and Will, 1986; Will, 2006), the current attempts to measure the Lense–Thirring effect (Lense and Thirring, 1918) on the planets' motions caused by the solar rotation (Iorio, 2005, 2012), and the Shapiro delay (Shapiro, 1964; Shapiro et al., 1971; Kramer et al., 2006)—the gravitational redshift is one of the experimental tests of GTR (Will, 2006).

Atom interferometry experiments can be used to measure the acceleration of free fall, see, for instance, Peters et al. (1999) and Müller et al. (2010a). The same research team has in the meantime argued that atom interferometry can also perform gravitational redshift measurements at the Compton frequency. This claim was criticized as incorrect by Wolf et al. (2010) leading to a response in support of the original result (Müller et al., 2010b). This controversy has continued until recently (Wolf et al., 2011, 2012; Hohensee et al., 2011, 2012; Hohensee and Müller, 2013).

2. Is there a physical process causing the redshift?

One aspect of the dispute between Müller et al. (2010a) and Wolf et al. (2010) is particularly disturbing and will be analysed here in some detail: Even after the prediction of the gravitational redshift by Einstein (1907) for over a century and the many observational confirmations mentioned in Section 1, there appears to be no consensus on the physical process (es) causing the shift. This can be exemplified by two conflicting statements. The first made by Wolf et al. (2010) reads:

> “The situation is completely different for instruments used for testing the universality of clock rates (UCR). An atomic clock delivers a periodic electromagnetic signal the frequency of which is actively controlled to remain tuned to an atomic transition. The clock frequency is sensitive to the gravitational potential U and not to the local gravity field $\boldsymbol{g} = \nabla U$. UCR tests are then performed by comparing clocks through the exchange of electromagnetic signals; if the clocks are at different gravitational potentials, this contributes to the relative frequency difference by $\Delta\nu/\nu = \Delta U/c^2$.”

Whereas in the second statement it is claimed by Müller et al. (2010b)[1]:

> “We first note that no experiment is sensitive to the absolute potential U. When two similar clocks at rest in the laboratory frame are compared in a classical red-shift test, their frequency difference $\Delta\nu/\nu = \Delta U/c^2$ is given by $\Delta U = \boldsymbol{g}\,\boldsymbol{h} + \mathcal{O}(h^2)$, where $\boldsymbol{g} = \boldsymbol{\nabla} U$ is the gravitational acceleration in the laboratory frame, $\boldsymbol{h}$ is the clock's separation, c is the velocity of light, and $\mathcal{O}(h^2)$ indicates terms of order h^2 and higher. Therefore, classical red-shift tests are sensitive to $\boldsymbol{g}$, not to the absolute value of U, just like interferometry red-shift tests.”

The potential at a distance r from a gravitational centre with mass M is constraint in the weak-field approximation for non-relativistic cases (Landau and Lifshitz, 1976) by

$$-c_0^2 \ll U = -\frac{G_N M}{r} \leqslant 0, \tag{2}$$

where G_N is Newton's constant of gravity. Wolf et al. (2010) could refer to many publications in their support (Einstein, 1907; von Laue, 1920; Schiff, 1960; Will, 1974; Okun et al., 2000; Sinha and Samuel, 2011). However, it would be required to define explicitly a reference potential U_0. A definition in line with Eq. (2) would give $U_0 = 0$ for $r = \infty$. Experiments on Earth (Pound and Rebka, 1959; Cranshaw et al., 1960; Hay et al., 1960; Krause and Lüders, 1961; Pound and Snider, 1965), in space (Bauch and Weyers, 2002) and in the Sun–Earth system (St. John, 1928; Blamont and Roddier, 1961; Brault, 1963; Snider, 1972; LoPresto et al., 1991; Cacciani et al., 2006; Takeda and Ueno, 2012) have quantitatively confirmed in this approximation a relative frequency shift of

$$\frac{\nu' - \nu_0}{\nu_0} = \frac{\Delta\nu}{\nu_0} \approx \frac{\Delta U}{c_0^2} = \frac{U - U_0}{c_0^2}, \tag{3}$$

where ν_0 is the frequency of a certain transition at U_0 and ν' the observed frequency there, if the emission caused by the same transition had occurred at a potential U. The question whether the shift happens during the emission process or is a result of a propagation effect is left open by Dicke (1960):

> “To return briefly to the question of the gravitational red shift, it is concluded that there could be two different red-shift effects. One would be interpreted in the usual way as a light propagation effect. The other, if it exists, would be interpreted as resulting from an intrinsic change in an atom with gravitational potential. The experiment employing an atomic clock in space would be one way of observing this effect directly, if it exists.”

There appears to be agreement, however, that the energy of a photon, $E_\nu = h\,\nu$, with Planck's constant h, does not vary during the propagation in a static gravitational field—excluding a variation of ν with changing U, if ν is measured against the coordinate or world time (Okun, 2000; Okun et al., 2000). This is consistent with the time dilation of atomic clocks derived from the GTR (Einstein, 1916) and, consequently, the matter would be settled, if geometric effects were considered to be an adequate cause of the gravitational redshift. Straumann (2000) discussed the modification of the electric potential by gravity in this context.

Wolf et al. (2010) and Müller et al. (2010b) have tried, however, to explore physical processes that cause the shift; yet both attempts are problematic in view of the fact that the gravitational force acting on the electron in transition is extremely small relative to the internal forces. This can easily be verified by a comparison of the weak solar gravitational force $\boldsymbol{K}_G^{\odot}$ acting on the electron in a hydrogen atom in the photosphere of the Sun with the electrostatic force $\boldsymbol{K}_E$:

$$\begin{aligned}\frac{\|\boldsymbol{K}_G^{\odot}\|}{\|\boldsymbol{K}_E\|} &= \frac{G_N M_{\odot} m_e}{R_{\odot}^2}\left(\frac{e^2}{4\pi\varepsilon_0 a_0^2}\right)^{-1} = \frac{r_S^{\odot}}{2R_{\odot}^2}\, m_e c_0^2 \left(\frac{e^2}{4\pi\varepsilon_0 a_0^2}\right)^{-1} \\ &= 3.031 \times 10^{-21}\end{aligned} \tag{4}$$

with $G_N = 6.674 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$; $M_{\odot} = 1.989 \times 10^{30}$ kg, the mass and $R_{\odot} = 6.960 \times 10^8$ m, the radius of the Sun; the mass of an electron $m_e = 9.109 \times 10^{-31}$ kg, $e = 1.602 \times 10^{-19}$ C, the elementary charge; $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F\,m^{-2}}$, the permittivity of the vacuum; $a_0 = 5.292 \times 10^{-11}$ m, the Bohr radius; and $r_S^{\odot} = 2G_N M_{\odot}/c_0^2 = 2950$ m, the Schwarzschild radius of the Sun.

The early attempts to measure the gravitational redshift of solar spectral lines as well as those of the white dwarf star Sirius B have been reviewed by Hetherington (1980). In particular, the wrong value of 21 $\mathrm{km\,s^{-1}}$ published by Adams (1925) has been contrasted with the result of $(89 \pm 16)\ \mathrm{km\,s^{-1}}$ obtained by Greenstein et al. (1971) for the companion of Sirius with $R/R_{\odot} = 0.0078 \pm 0.0002$ and $M/M_{\odot} = 1.20 \pm 0.25$. These radius and mass data inserted into Eq. (4) instead of the solar values give 5.9×10^{-17}. Mean gravitational redshifts of $(53 \pm 6)\ \mathrm{km\,s^{-1}}$ for six white dwarfs in the Hyades have been measured by Greenstein and Trimble (1967).

Even for the very strong gravitational field of the neutron star EXO 0748-676, for which Cottam et al. (2002) found a redshift of $z = 0.35$ in Fe XXVI and Fe XXV as well as in O VIII lines, a calculation similar to Eq. (4) yields

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[1] In this quotation the expression h^2 indicates the square of the clock separation $\boldsymbol{h}$ and is not related to Planck's constant h used below.