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Kaluza Klein universe with magnetized anisotropic dark energy in general relativity and Lyra manifold

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HIGHLIGHTS

- The magnetic field isotropizes the distribution of dark energy.
- The universe approach to isotropy monotonically even in the presence of the anisotropic fluid.
- Results are consistent with cosmological observations.
- The concept of Lyra manifold does not remain for a very large time.
- It is interesting to note that the higher the anisotropy of the expansion, the higher the energy density.

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ABSTRACT

In this paper, the authors have investigated the Kaluza Klein universe with magnetized anisotropic dark energy in the context of Lyra manifold. Exponential and power law volumetric expansion is assumed to obtain the solution of the field equations. It is observed that magnetic field plays significant role in isotropization of the dark energy. The physical parameters of the models have been discussed in detail.

1. Introduction

In General Relativity theory, Einstein for the first time geometrized the gravitation. Many experimental phenomenons assert the proof for theoretical explanation. Weyl in 1918 was inspired by it and he was the first to unify gravitation and electromagnetism in single space–time geometry. He showed how one can introduce a vector field in the Riemannian space–time with an intrinsic geometrical significance. But this theory was not accepted as it was based on non-integrability of length transfer. Lyra (1951) introduced a gauge function i.e. a displacement vector in Riemannian space time which removes the non-integrability condition of a vector under parallel transport. In this way Riemannian geometry was given a new modification by him and the modified geometry was named as Lyra manifold (Agarwal et al., 2011). Sen (1957) developed a new scalar tensor theory of gravitation and constructed an analogue of Einstein's field equation based in Lyra geometry. Halford (1970) has shown that scalar tensor theory of gravitation in Lyra manifold predicts the same effect as in Einstein's general relativity and the displacement vector of Lyra geometry plays the role of cosmological constant in general relativity. On the basis of Lyra geometry with time dependent cosmological model is studied by Singh and Singh (1992). Agarwal et al. (2011) investigated the LRS Bianchi type II perfect fluid cosmological models in normal gauge for Lyra manifold. Pradhan et al. (2003, 2006, 2007), Casama et al. (2006), Bali and Chandnani (2008), Kumar and Singh (2008), Ram et al. (2008), Singh (2008) and Rao et al. (2008) has studied cosmological models based on Lyra geometry in various contexts.

Observations such as cosmic microwave background, Supernovae (Riess et al., 1998; Tonry et al., 2003) have indicated that the expansion of the universe has entered a phase of acceleration. The recent extensive search for a matter field has given rise to the universe (Kusenko, 1998). This type of matter is called Q-matter. At

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the present epoch, a lot of works has been done to solve this candidate for Q-matter that has so far been a scalar field having a potential which generates a sufficient negative pressure (Chattopadhyay, 2010). To explain the nature of dark energy many models have been proposed such as quintessence (Sami, 2003), k-essence (Chiba, 2002), tachyon (Sen, 1999), phantom (Caldwell, 2002), and Chaplygin gas (Kamenshik, 2001).

In recent time, Rao et al. (2011) and Naidu et al. (2012a), have discussed dark energy models of Bianchi type I and II respectively in Saez–Ballester theory of gravitation. Bianchi type III dark energy model in a Saez–Ballester scalar tensor theory of gravitation is explained by Naidu et al. (2012b). Katore et al. (2012) investigated Bianchi type III dark energy model in Scalar tensor theory of gravitation. Singh and Sharma (2014) studied Bianchi type II cosmological model with dark energy in scale covariant theory of gravitation. Kaluza Klein cosmological models with anisotropic dark energy are investigated by Adhav et al. (2011).

Magnetism plays an important role in structure formation and also affects the anisotropies in the cosmic microwave background radiation. We observe magnetic fields in stars, galaxies, clusters and high red shift Lyman system. Magnetic field originates either in the early universe and already present at the onset of structure formation or generated by battery mechanism during the initial stages of structure formation. The relation between magnetic field and matter is one of the attracting area of research. Magnetized quark and strange quark matter in the spherical symmetry space time one parameter group of conformal motions is examined by Aktas and Yilmaz (2007). Yadav et al. (2012) investigated magnetized dark energy and late time acceleration. A new class of exact solution of Einstein's modified field equations in inhomogeneous space time for perfect fluid distribution with variable magnetic permeability and time dependent gauge function within the framework of Lyra geometry is investigated by Singh and Singh (2012). Saha and Visinescu (2008) obtained some exact solution of the Bianchi type I massive string cosmological model in the presence of a magnetic field. Sharif and Zubair (2010) studied the Bianchi type I cosmological model in the presence of magnetized anisotropic dark energy. This motivates us to investigate Kaluza Klein universe with magnetized dark energy in General Relativity and Lyra manifold

2. Metric and field equation

We consider the Kaluza–Klein metric

$$ds^2 = -dt^2 + A(t)^2(dx^2 + dy^2 + dz^2) + B^2 d\Phi^2, \tag{1}$$

where A and B are functions of time t only. The dimensionality of the world has long been a subject of discussion. Because we perceive only four dimensions, but in general relativity there is nothing in the equation which restrict us to four dimensions. Cosmologist believes that the universe might have a higher dimensional era at its early stage of evolution. The higher dimensional theory proposed by Kaluza (1921) by unifying Einstein's theory of General Relativity with Maxwell's theory of Electromagnetism in a five dimensional manifold. He thought that our world may have more than four dimensions. Klein (1926) has elegant presentation in terms of geometry. He treated the extra dimension as compact small circle topologically. It is five dimensional, but it looks like ordinary gravity in free space. Cosmological and astrophysical implications of extra dimension have been discussed by Overduin and Wesson (1997). To understand the behavior of quintessence, the existence of extra dimensions is necessary. As our real universe is four dimensional, the hidden dimensions must be related to the dark energy and dark matter which are also not visible. In Space–Time–Matter theory, it is proposed that hypersurface embedded in the 5D Ricci flat manifold and all the matter in our world are induced from higher dimension. Chodos and Detweiler (1980) suggested that in Kaluza–Klein theory the extra dimension contracts to a very small scale, while the three other spatial dimensions expand isotropically. During this contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario (Guth, 1981; Alvarer et al., 1983). Quark matter coupled to the string cloud and domain wall in five dimensional Kaluza Klein space time is studied by Yilmaz (2006).

The energy momentum tensor of fluid is taken as

$$\begin{aligned} T_j^i &= diag[T_1^1, T_2^2, T_3^3, T_4^4, T_5^5] \\ &= diag[p_x - \rho_B, p_y - \rho_B, p_z + \rho_B, p_\Phi + \rho_B, -\rho + \rho_B] \\ &= diag[\omega_x\rho - \rho_B, \omega_y\rho - \rho_B, \omega_z\rho + \rho_B, \omega_\Phi\rho + \rho_B, -\rho + \rho_B], \end{aligned} \tag{2}$$

Where ρ is the energy density; p_x, p_y, p_z, p_Φ are pressure on x, y, z and Φ axes respectively; ρ_B stands for energy density of magnetic field where as $\omega_x, \omega_y, \omega_z, \omega_\Phi$ are the directional EOS parameter along x, y, z and Φ axes respectively. By setting $\omega_x = \omega + \delta, \omega_y = \omega + \gamma, \omega_z = \omega + \eta, \omega_\Phi = \omega$

We have

$$\begin{aligned} T_j^i = diag[&(\omega+\delta)\rho - \rho_B, (\omega+\gamma)\rho - \rho_B, (\omega+\eta)\rho + \rho_B, \omega\rho \\ &+ \rho_B, -\rho + \rho_B], \end{aligned} \tag{3}$$

where ω is deviation free parameter and δ, γ, η are the skewness parameter. If the deviation parameters are zero, then Eq. (2) represents the energy momentum tensor for the isotropic fluid and magnetic field. For zero magnetic field Eq. (2) reduces to the energy momentum tensor of anisotropic fluid (Adhav et al., 2011). The effect of magnetic field on the evolution of the universe using magnetized perfect fluid energy momentum tensor discussed by Jacobs (1969), King and Colesb (2007). The field equations (in gravitational units $c = 1 = 8\pi G$) in normal gauge for Lyra manifold were obtained by.

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\Phi_i\Phi_j - \frac{3}{2}g_{ij}\Phi_k\Phi^k = -T_{ij}, \tag{4}$$

where Φ is the displacement field vector defined as $\Phi_i = (0, 0, 0, 0, \beta(t))$. The other symbols have their usual meaning as in Riemannian geometry. Soleng (1987) investigated that the cosmologies based on Lyra's manifold with constant gauge vector Φ and a creation field becomes Hoyle's creation field cosmology or contains a special vacuum field, which together with the gauge vector term may be considered as a cosmological term (Hoyle, 1948; Hoyle and Narlikar, 1963, 1964). Beesham (1986a) conclude that, the advantage of cosmological model based on Lyra's manifold rather than a Riemannian manifold is that the cosmological constant arises naturally from the geometry rather than being introduced in an arbitrary adhoc fashion. Recently, Nagpure (2013) studied Kaluza–Klein cosmological model in Lyra geometry.

In co-moving coordinate system field equations for the metric (1) with the help of (3) take the form

$$\frac{A_5^2}{A^2} + \frac{2A_{55}}{A} + \frac{B_{55}}{B} + \frac{2A_5B_5}{AB} + \frac{3}{4}\beta^2 = -(\omega+\delta)\rho + \rho_B, \tag{5}$$

$$\frac{A_5^2}{A^2} + \frac{2A_{55}}{A} + \frac{B_{55}}{B} + \frac{2A_5B_5}{AB} + \frac{3}{4}\beta^2 = -(\omega+\gamma)\rho + \rho_B, \tag{6}$$

$$\frac{A_5^2}{A^2} + \frac{2A_{55}}{A} + \frac{B_{55}}{B} + \frac{2A_5B_5}{AB} + \frac{3}{4}\beta^2 = -(\omega+\eta)\rho - \rho_B, \tag{7}$$

$$\frac{3A_5^2}{A^2} + \frac{3A_{55}}{A} + \frac{3}{4}\beta^2 = -\omega\rho - \rho_B, \tag{8}$$

$$\frac{3A_5^2}{A^2} + \frac{3A_5B_5}{AB} - \frac{3}{4}\beta^2 = \rho - \rho_B. \tag{9}$$