New Astronomy 28 (2014) 54–62

Contents lists available at ScienceDirect

New Astronomy

journal homepage: www.elsevier.com/locate/newast

Dynamics and gravitational waveforms in spinning compact eccentric binaries

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HIGHLIGHTS

- We numerically study the effect of eccentricity on the PN spinning compact binaries.
- The eccentricity has a great effect on the dynamics due to gravitational radiation.
- The dynamics and the eccentricity jointly modulate the gravitational waveforms.
- The spin–spin effect plays an important role during the gravitational radiation.
- The imprint can be deduced from the time-domain and frequency-domain waveforms.

ARTICLE INFO

Article history:
Received 8 June 2013
Received in revised form 8 October 2013
Accepted 9 October 2013
Available online 17 October 2013

Communicated by E.P.J van den Heuvel

Keywords:
Celestial mechanics
Methods: spinning compact binary simulations
Methods: gravitational waves
Methods: chaos

ABSTRACT

The effects of eccentricity on the Hamiltonian dynamics of post-Newtonian spinning compact binaries and gravitational radiation from eccentric orbits are discussed. The simulation results of scans for chaos show that the eccentricity has a great effect on the dynamics without considering dissipation due to gravitational radiation. Moreover, both the dynamics behavior and the orbital eccentricity jointly modulate the gravitational waveforms, and the spin–spin coupling effect play an important role during the gravitational radiation of inspiral and coalescence. Moreover, the imprint of characteristic of the original system can be deduced from the time-domain and frequency-domain waveforms.

1. Introduction

The gravitational waves emitted from the spinning compact binaries moving in general orbits are one of the most promising sources for the gravitational-wave detectors. The match data analysis of gravitational waves are strongly dependent on the theory template besides the azimuth of observer. A number of works on analysis of gravitational waves of spinning compact binaries have been done by using the post-Newtonian (PN) gravitational circular waveform templates (Kidder, 1995; Will and Wiseman, 1996; Buonanno et al., 2006; Blanchet et al., 2008), where eccentricity is neglected. Under the dissipative effect of gravitational radiation reaction, the two body separation r damp down during inspiral and coalescence, the binary orbit gradually tends to a quasi-circular actually the residual eccentricity still affect the detection of gravitational waves (Levin et al., 2011; Kidder and Zimmerman, 2010; Csizmadia et al., 2012), of course, if the residual eccentricity is on the verge of zero, it's effect on gravitational waves can be negligible. So it is significant to engage in the researches on the gravitational waves emitted from spinning eccentric binaries.

Moreover, the presence of chaos can improve the amplitude of gravitational waves (Suzuki and Maeda, 1999). The highly eccentric orbits preferred to be chaotic due to the large velocities in the post-Newtonian Hamiltonian of spinning compact binaries (Hartl and Buonanno, 2005), but the onset of chaos can appear in the lowly eccentric zone for the spinning compact binaries described by the PN Lagrangian (Wu and Xie, 2008). In this sense, the dynamics analysis of spinning eccentric binaries relies on not only the PN approximations approach, the dynamical parameters, and the initial conditions of spinning compact binaries, but also the high-precision integrator. When symplectic integrators with constant step size applied to a high eccentric orbit or close encounters with objects, it cannot effectively preserve the geometrical

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1384-1076/$ - see front matter
http://dx.doi.org/10.1016/j.newast.2013.10.004

structure of Hamiltonian systems in the longtime integration and lead to the numerical integration results deviation, some discussions associated with these results were given in Refs. Gladman et al. (1991), Preto and Tremaine (1999), Hut et al. (1995), Hairer (1997). When we study the high eccentric PN binary system, how to improve the numerical stability affected by the spin effects and orbital eccentricity? Along the above references direction, we control effectively the longtime integration error linear growth and keep the stability of symplectic integrator by introducing the spin parameters into the monitor function in the form of time transformations, the detailed implementation as shown in reference Zhong and Liu (2012).

In this paper, we numerically investigate the dynamics of spinning compact binaries moving in a high eccentric orbit by using the adaptive time-step leap frog integrator. The rest of this paper is organized as follows. In Section 2, PN Hamiltonian formula of physical model are introduced concisely. Then we test the numerical precision and stability of symplectic integrator with adaptive time-step. Section 3 the different scans for chaos and order are given by virtue of the fast Lyapunov indicator (FLI). Section 4 mainly focus on evaluating the gravitational waves from spinning eccentric binaries. Finally, Section 5 summarizes our conclusions. Geometric units $c = G = 1$ are used throughout the work.

2. Physical model and integration testing

2.1. The 3PN Hamiltonian formulation

In this section, we use the PN Hamiltonian formulation describing equations of motion of spinning eccentric binaries (Damour et al., 2000; Damour, 2001; Buonanno et al., 2006). In the center of mass frame, the dimensionless conservative PN Hamiltonian with non-canonical spins variables for the relativistic two-body with masses m_1 and m_2 can be written as

$$H = H_N + H_{(1PN,2PN,3PN)} + H_{2.5PN} + H_{SO} + H_{SS}, \tag{1}$$

where H_N, H_{PN}, H_{SO} and H_{SS} denote the Kepler flow, the orbital PN corrections, 1.5PN spin–orbit coupling term and the 2PN spin–spin coupling term, respectively.

On the other hand, the spin effects part H_{SO} and H_{SS} have the form

$$H_{SO} = \frac{1}{r^3}\mathbf{L}\cdot\mathbf{S}_{eff}, \tag{2}$$

$$H_{SS} = \frac{1}{2r^3}[3(\mathbf{S}_0\cdot\mathbf{n})^2 - \mathbf{S}_0^2], \tag{3}$$

here $\mathbf{L} = \mathbf{r}\times\mathbf{p}$, $(\mathbf{r},\mathbf{p})$ are conjugate position and conjugate momentum. The other notations are specified as

$$\mathbf{S}_{eff} = \left(2 + \frac{3m_2}{2m_1}\right)\mathbf{S}_1 + \left(2 + \frac{3m_1}{2m_2}\right)\mathbf{S}_2, \tag{4}$$

$$\mathbf{S}_0 = \left(1 + \frac{m_2}{m_1}\right)\mathbf{S}_1 + \left(1 + \frac{m_1}{m_2}\right)\mathbf{S}_2, \tag{5}$$

with dimensionless spin variables are defined as

$$\mathbf{S}_i = \chi_i m_i^2/M^2\widehat{\mathbf{S}}_i, \tag{6}$$

where $\widehat{\mathbf{S}}_i$ represents a unit spin vector, and dimensionless parameters $\chi_i \in [0,1]$.

The equations of motion about canonical variables $(\mathbf{r},\mathbf{p})$ including 2.5PN dissipation due to gravitational radiation reaction is

$$\frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial\mathbf{p}},\quad \frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial\mathbf{r}} + \mathbf{F}, \tag{7}$$

here $\mathbf{F}$ denotes a dissipation force, namely radiative contributions to the accelerations, the expression of accelerations in term of variables $(\mathbf{r},\mathbf{p})$ can be found in Ref. (Levin et al., 2011).

Moreover, the two spins evolve as

$$\frac{d\mathbf{S}_1}{dt} = \frac{\partial H}{\partial\mathbf{S}_1}\times\mathbf{S}_1,\quad \frac{d\mathbf{S}_2}{dt} = \frac{\partial H}{\partial\mathbf{S}_2}\times\mathbf{S}_2. \tag{8}$$

For an illustration, $\mathbf{r},\mathbf{p}$ and $\mathbf{S}_i$ all are vectors in $\mathbb{R}^3$ in the above equations, among them, the position and momentum coordinates $(\mathbf{r},\mathbf{p})$ are a pair of conjugate variables, but the spin coordinates are not, the reasons can be found in Ref. (Wu and Xie, 2010). From the view of the globally symplectic structure, the spin evolution equations can't be integrated numerically by the symplectic integrator. But according to the method of coordinate transformation provided in Ref. Wu and Xie (2010), the trouble can be easily expelled, then each spin has the following transformation

$$\mathbf{S}_i : (S_{i1}, S_{i2}, S_{i3}) \in \mathbb{R}^3 \to (\theta_i, \xi_i) \in \mathbb{R}^2, \tag{9}$$

here each spin is viewed as a two-dimensional vector with 3 components, θ_i is a spin azimuth angle, ξ_i is the spin z component. In Ref. Wu and Xie (2010), it presents sufficient reasons to explain why the variables (θ_i, ξ_i) are conjugate spin coordinates, then the spin Hamiltonian part can be re-expressed by using conjugate spin variables (θ_i, ξ_i), so Eq. (1) rewritten as

$$\widetilde{H}(\mathbf{r},\theta;\mathbf{p},\xi) = H_N + H_{PN} + \widetilde{H}_{SO} + \widetilde{H}_{SS}. \tag{10}$$

Therefore, it presents a good chance to apply symplectic integrator for the PN Hamilton of spinning compact binaries. In the following, we will numerically investigate the dynamics of post-Newtonian Hamiltonian formulations for eccentric spinning compact binaries.

2.2. Integration test for high eccentric spinning compact binary

On the basis of the perturbation splitting of Hamiltonian formulism, we design the global preserving symplectic structure integrators so as to study the dynamics of conservative PN spinning compact binaries (Zhong et al., 2010), in the cases of lower eccentric orbits, the integrators have excellent secular behavior, but the symplectic geometric structure of system will be broken if the integrators applied to high eccentric binaries. In our later work (Zhong and Liu, 2012), in term of expanding phase space of PN Hamiltonian, we developed the adaptive time-step symplectic integrators (ASI) to the high eccentric spinning binaries.

When the given Hamiltonian (10) with radiation reaction cut off, can be split two separable part

$$\widetilde{H}(\mathbf{r},\theta;\mathbf{p},\xi) = H_N + H_{pert}, \tag{11}$$

where

$$H_{pert} = H_{PN} + H_{2PN} + H_{3PN} + \widetilde{H}_{SO} + \widetilde{H}_{SS}, \tag{12}$$

by adding a auxiliary variables ψ to Hamiltonian (11), the 10-dimensions phase space of system (11) change into 12-dimensions, therefore, besides $(\mathbf{r},\mathbf{p})$ and (θ,ξ), original time t and ψ are also viewed as a pair of canonical conjugate variables.

Then adaptive time step integrator

$$ASI = e^{\frac{t}{2}\psi} e^{\frac{t}{2\psi}\widehat{H}_N} e^{\frac{t}{\psi}\widehat{H}_{pert}} e^{\frac{t}{2\psi}\widehat{H}_N} e^{\frac{t}{2}\psi}, \tag{13}$$

the symbol $\widehat{H}$ denotes the Lie derivative in method ASI.

In addition, the fixed time step mixed leapfrog integrator is

$$SI = e^{\frac{t}{2}\widehat{H}_N} e^{t\widehat{H}_{pert}} e^{\frac{t}{2}\widehat{H}_N}. \tag{14}$$