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An analytic function of the two-dimensional probabilities of perception of the human eyes

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ABSTRACT

The probability of perception is an indispensable quantity in the visual observation of meteors. Based on the data of a large number of double-counting observations, Koschack and Rendtel derived a table-listed average perception function $P(\Delta m)$ in 1990 and Wu gave it a fitting analytic function in 2005. In this paper, a fitting of the perception function in the two-dimensional field of view, $P(\Delta m, R)$, is given. Both the new analytic function and each order of its derivatives have only a monodromy and are very smooth. This analytic function will be more essential and useful than the average function $P(\Delta m)$ and may be connected to the two-dimensional structure of the human eyes as an imaging system.

1. Introduction

The macroscopic observation is the first way to understand the meteors and meteor showers in ancient years, including some special meteoric phenomena recorded in ancient Chinese documents and others (Wu and Zhang, 2003; Yang et al., 2005; Zhuang, 1977). Nowadays, several advanced techniques have been developed in modern meteor astronomy, but the visual observation is still the most popular and most fundamental (Arlt et al., 1999; Arlt and Rendtel, 2006; Gural, 2004; Wu, 2005; Wu and Li, 2003). In recent years, we have also done several pieces of research work based on the visual observation of meteors (Li et al., 2002; Li and Zhao, 2002; Wu, 2006; Wu et al., 2001; Wu and Zhang, 1998, 2002), despite some pieces of work on photographic (Wu, 2007; Wu and Zhang, 2006; Zhang and Wu, 2002) or video observation (Wu and Zhang, 2004), but without spectroscopic (Koten et al., 2006) and radar observations (Campbell-Brown et al., 2006).

In recent decade, the greatest progress might be the prediction of meteor showers (McNaught and Asher, 1999; Lyytinen and Van Flandern, 2000; Vaubaillon et al., 2005). However, modern meteor astronomy has had only a short history since a strong Leonid meteor storm happened in 1833 (Rao, 1998, 1999). Modern sciences and technologies also have a shorter history, even we human beings have not understood ourselves thoroughly, including our eyes.

The human eye is a very subtle camera, which has two kinds of visual cells. One has high resolving power for the days and another has high sensitivity for the nights. However, the meteor phenomenon occurs in the Earth's atmosphere at random. An individual meteor moves fast and normally stays in the sky for less than one second. Therefore, you could not catch every meteor which appeared in your field of view. Some meteors are in the shoulder of your responses of the luminous sensitivity and the perceptive speed, only a fraction of the meteors can be visible to the naked eye. The number of the observed meteors at a certain magnitude m can be written as

$$N_{m,\mathrm{Obs}} = P(m) \times N_m, \quad (1)$$

where N_m is the true number of the meteors at this magnitude m and $P(m)$ is the so-called "probability of perception". It is known that for every person the faintest magnitude must exist, which can be called the "limiting stellar magnitude LM" for an individual observer. Strictly speaking, the probability $P(m)$ depends upon the limiting magnitude LM, the angular distance R from the center of the viewing field, the elevation of the center of the field h_f, the

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angular velocity of the meteor and the magnitude distribution of the meteors, etc. However, it is impossible to measure the true probability for each observer. In this aspect, Koschack and Rendtel (1990a,b) gave a lot of measurements and detailed discussions. They made a "standard" model of the perception as a function of Δm, which is the magnitude difference

$$\Delta m = \mathrm{LM} - m. \tag{2}$$

In addition, they proved that a deviation from the standard perception function for an individual person could be expressed by an error of the limiting magnitude, $\Delta\mathrm{LM}$, and can be regarded as a correction (Koschack and Rendtel, 1990a,b). Over the years, their suggestion has become a standard method for the International Meteor Organization (IMO for short) and a lot of observers of the visual meteors (Arlt et al., 1999; Brown, 1990; Jenniskens, 1994; Li et al., 2002; Watanabe et al., 1999; Wu, 2005; Wu and Li, 2003).

In this paper, we will further research the "probability of perception" or "perception function", since it is a very important parameter not only in the meteor astronomy, but also in the structure of the human eyes.

2. The fitting of the average perception function $P(m)$ of the human eyes

2.1. The fitting by a sine function

Based on a large number of samples of meteors observed in the modified double-counting observation, Koschack and Rendtel (1990b) derived a table-listed average perception function $P(\Delta m)$, which has been widely used in circles of meteor astronomy. However, a list of data is usually not as convenient as an analytic formula for both the calculation and analysis. In our first paper (Wu and Li, 2003), we tried to fit their data using a sine function like this

$$P(\Delta m) = 0.5 + 0.5 \times \sin\left\{\frac{\pi}{2}\left[\left(\frac{\Delta m}{c}+q\right)^p - \left(\frac{3.93}{c}+q\right)^p\right]\right\}, \tag{3}$$

where π is taken as 3.1416. This perception function looks fair with a root-mean-square error (*rms* for short) of 0.008. However, it needs to fit the data independently in two sections and the two groups of constants are obtained as follows:

$$p = 2.7, \quad q = 0.38, \quad c = 6.33, \ (\text{for } \Delta m \leqslant 3.93), \tag{4}$$

$$p = 0.78, \quad q = -0.80, \quad c = 4.90, \ (\text{for } \Delta m > 3.93). \tag{5}$$

In addition, one of its greatest defects lies in that, in some cases, two maxima might exist in the magnitude distribution curve of the meteors, since there is one discontinuity in the derivative curve of the fitted $P(\Delta m)$ (Wu, 2005).

2.2. The uniform fitting of a function of hyperbolic tangent

A function formed by the hyperbolic tangent was suggested in our second paper (Wu, 2005), which can be expressed as

$$P(\Delta m) = 0.5 + 0.505 \times \tanh(0.66 \cdot \Delta m - 0.013 \cdot \Delta m^2 - 2.43). \tag{6}$$

It seems that the new fitting has rather greater *rms* error than the old one. However, you had better see the relative errors. In the ordinary way, we have

$$\log\frac{P_{\mathrm{F}}(\Delta m)}{P_{\mathrm{T}}(\Delta m)} = \log(P_{\mathrm{F}}(\Delta m)) - \log(P_{\mathrm{T}}(\Delta m)), \tag{7}$$

where $P_{\mathrm{F}}(\Delta m)$ is the fitting value of the average perception function $P(\Delta m)$ and $P_{\mathrm{T}}(\Delta m)$ is the true value. We hope that the fitting is perfect and the logarithmic difference is near zero at every point of Δm. The fitting of Eq. (6) has a *rms* error of 0.023 in the logarithmic scale for the data of Koschack and Rendtel (1990b) in the range of $-0.2 \leqslant \Delta m \leqslant 7.6$, which is much less than the old one of about 0.045 in the logarithmic scale. It means that this fitting has only a relative deviation as small as about 2% and is more consistent in the whole range. Moreover, not only the new perception function has not any discontinuity everywhere, but also the first and Nth derivatives are continuous and smooth.

The fitting of the perception probabilities seems to be finished, but we wish to simplify the fitting function and especially to discuss the relationship between the two-dimensional perception probability and the structure of the eyes as an imaging system.

2.3. Linear Tanh – a simplified fitting of the average perception function

Inside the hyperbolic tangent of Eq. (6) there is a square item of Δm. However, it can be seen that while Δm is as large as 8, we have $0.66 \cdot \Delta m = 5.28$ and $0.013 \cdot \Delta m^2 = 0.832$. It means, in all cases that the relationship of $0.66 \cdot \Delta m \gg 0.013 \cdot \Delta m^2$ always exists. This fact indicates that the square item can be ignored and the function can be changed into

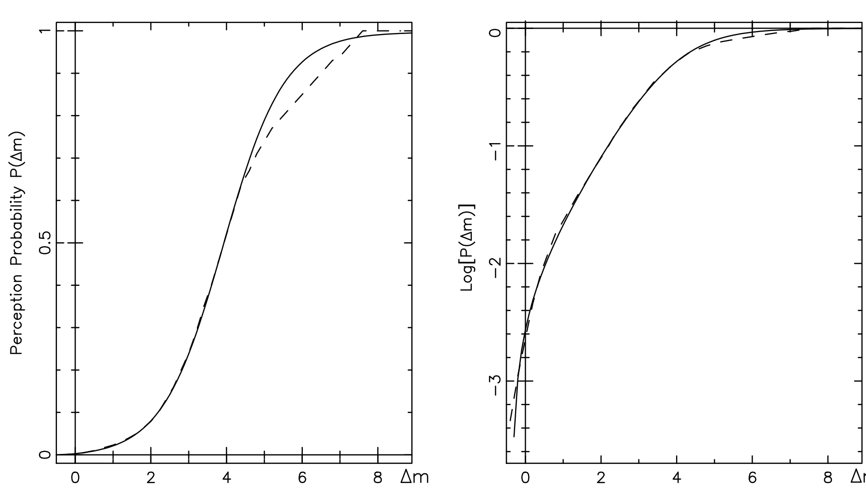

Fig. 1. A newly simplified fitting to the data of the perception probabilities listed by Koschack and Rendtel (1990b), (indicated by the dashed line): (a) in the linear scale and (b) in the logarithmic scale.