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Solitons in large scale structures

A.L.B. Ribeiro *

Laboratório de Astrofísica Teórica e Observacional, Departamento de Ciências, Exatas e Tecnológicas, Universidade Estadual de Santa Cruz, Rodovia Ilhéus-Itabuna km 16, 45662.000 Ilhéus, Brazil

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Abstract

I propose an alternative approach to extend the scope of the analytical understanding of structure formation in the universe. By assuming a scalar field ψ whose Lagrangian density implies a sine-Gordon equation for the unstable modes, we find the standard 1D solution for this equation over the nonlinear regime. The solutions predict the appearance of periodic nonlinear waves with soliton features in the cosmic fluid. I also introduce a procedure to transform ψ to the density matter field ρ and present some simple profiles of the resulting structures.

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1. Introduction

Structure formation in the universe can be described in the context of fluid dynamics. The usual approach is to perturb the hydrodynamics equations and follow the evolution of perturbations until the modes enter into the nonlinear regime (e.g. Peebles, 1980). This method is restricted to the early stages of the entire process, which imposes a limit for the analytic understanding on how structures are formed. In the nonlinear regime, N-body simulations with increasing particle resolution are used to access the complex emergence of structure patterns. However, these numerical studies do not unveil the specific gravitational collective phenomena operating behind the resulting hierarchy of structures. In this context, attempts to extend the scope of analytical models to the nonlinear regime are welcome. With this respect, important developments have been done in recent years (e.g. Jones, 1999; Coles and Spencer, 2003). Following this research line, I propose a complementary approach to probe some aspects of the nonlinear regime of cosmic structure formation. The basic idea is to define a mapping between the density fluctuations and a generic scalar field whose dynamical equation is equivalent to that we obtain from fluid dynamics. In particular, I am interested in solutions to the dynamics over the nonlinear regime, where nonlinear waves could develop in the cosmic fluid.

2. The field approach

I consider a generic instability scenario in which perturbations are generated by some mechanism in the early stages of the universe evolution, and start to significantly grow after recombination. The density of matter can be described by $\rho(x, t) = \rho_b[1 + \delta(x, t)]$, with dynamical evolution for δ given by

$$\ddot{\delta}_n + 2H\dot{\delta}_n + \left(\frac{v^2k_n^2}{a^2} - 4\pi G\rho_b\right)\delta_n = 0 \qquad (1)$$

* Tel.: +55 73 3680 5167; fax: +55 73 3680 5230.
E-mail address: albr@uesc.br

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(e.g. Peebles, 1980). I propose in this paper that the density of matter (i.e. the density fluctuations field) can be associated to a general scalar field

$$\rho = \rho[\delta(x,t)] \rightarrow \psi(x,t) \tag{2}$$

whose Lagrangian density is

$$\mathscr{L} = \frac{1}{2}\left[(\partial_t\psi)^2 - v^2(\nabla\psi)^2\right] - U(\psi). \tag{3}$$

The corresponding momentum function is

$$\pi = \frac{\partial\mathscr{L}}{\partial(\partial_t\psi)} = \partial_t\psi \tag{4}$$

and hence the Hamiltonian density is

$$\mathscr{H} = \pi^2 - \mathscr{L} = \frac{1}{2}\left[\pi^2 - v^2(\nabla\psi)^2\right] - U(\psi). \tag{5}$$

The canonical equations are

$$\partial_t\psi = \pi \quad \text{and} \quad \partial_t\pi = -U'(\psi) + v^2\nabla^2\psi. \tag{6}$$

From these equations, we find the field equation

$$(\partial_t^2 - v^2\nabla^2)\psi + U'(\psi) = 0. \tag{7}$$

I assume the field is modelled as the limit of an infinite chain of coupled one-dimensional oscillators, whose potential energy function about the equilibrium point can be expanded in a Taylor series

$$U(\psi) = U_0 + U_0^{(1)}\psi + \frac{1}{2!}U_0^{(2)}\psi^2 + \frac{1}{3!}U_0^{(3)}\psi^3 + \frac{1}{4!}U_0^{(4)}\psi^4 + \cdots, \tag{8}$$

where U_0 is the value of $U(\psi)$ at the equilibrium point and $U_0^{(n)}$ is the nth-order derivative of the function at that point. For the special case of a symmetric potential energy, the expansion is just a sum of even-order terms and a natural choice for the function is the sine-Gordon potential $U_{sG}(\psi) = \Omega^2(1 - \cos\psi)$, whose Lagrangian density is

$$\mathscr{L}_{sG} = \frac{1}{2}\left[(\partial_t\psi)^2 - v^2(\nabla\psi)^2\right] - 2\Omega^2(1 - \cos\psi). \tag{9}$$

Hence, the dynamical equation becomes

$$(\partial_t^2 - v^2\nabla^2)\psi + \Omega^2\sin\psi = 0. \tag{10}$$

2.1. The linear regime

In order to use this model, we should introduce some constraint into the problem. The most obvious restriction is that we have to recover the dynamical equation for the density fluctuations over the linear regime. Note that, in the linearized limit, the sG equation becomes the Klein–Gordon equation and the motion equation is

$$(\partial_t^2 - v^2\nabla^2)\psi + \Omega^2\psi = 0. \tag{11}$$

with $U_{KG} = \Omega^2\psi^2$. Without loss of generality, on a finite one-dimensional interval $0 \leqslant x \leqslant X$, the Fourier decomposition of the field writes

$$\psi(x,t) = X^{-1/2}\sum_{-\infty}^{\infty}\delta_n(t)\,e^{ik_nx}. \tag{12}$$

If ψ is to be real, $k = -k_{-n}$, which implies that $\delta_n = \delta_n^*$. The Lagrangian is obtained by first writing $\mathscr{L}_{KG}$

$$\mathscr{L}_{KG} = \frac{1}{2X}\sum_{n,m}[\dot{\delta}_n\dot{\delta}_m + (v^2k_nk_m - \Omega^2)\delta_n\delta_m]\,e^{i(k_n+k_m)x}. \tag{13}$$

The integral in x is

$$L_{KG} = \frac{1}{2}\sum_n\dot{\delta}_n^*\dot{\delta}_n - \frac{1}{2}\sum_n(v^2k_n^2 + \Omega^2)\delta_n^*\delta_n, \tag{14}$$

where we have used the condition $\delta_n^* = \delta_{-n}$. The corresponding Euler–Lagrange equations are

$$\ddot{\delta}_n + (v^2k_n^2 + \Omega^2)\delta_n = 0. \tag{15}$$

These equations are uncoupled. In the special case $\Omega^2 = 0$, these are the equations for a set of uncoupled harmonic oscillators, each at its own frequency $\omega_n = vk_n$. Otherwise, in the case $\Omega^2 \neq 0$, we still have uncoupled equations, but with frequencies given by the dispersion relation

$$\omega_n = \sqrt{v^2k_n^2 + \Omega^2}. \tag{16}$$

Setting comoving instead of proper coordinates, we find an extra term in Eq. (15) due to the expansion of the universe

$$\ddot{\delta}_n + 2H\dot{\delta}_n + \left(\frac{v^2k_n^2}{a^2} + \Omega^2\right)\delta_n = 0. \tag{17}$$

where $H = \dot{a}/a$ is the Hubble parameter and $a(t)$ is the scale factor. This is the same we would find if we perturbed and linearized the standard fluid equations and set $\Omega^2 = -4\pi G\rho_b$ (see Eq. (1))

$$\ddot{\delta}_n + 2H\dot{\delta}_n + \left(\frac{v^2k_n^2}{a^2} - 4\pi G\rho_b\right)\delta_n = 0, \tag{18}$$

where $\omega_n^2 = v^2k_n^2 - 4\pi G\rho_b$ and ρ_b is the unperturbed background density. The condition $\omega_n^2 = 0$ defines the so-called Jeans wavelength:

$$\lambda_J = v\left(\frac{\pi}{G\rho_b}\right)^{\frac{1}{2}}. \tag{19}$$

Perturbations on scales $\lambda < \lambda_J$ oscillate as acoustic waves with a steadily decreasing amplitude, while perturbations on scales $\lambda > \lambda_J$ remain unaffected by pressure forces and continue to grow. When the amplitudes of these modes reach a value $\delta \sim 1$, they enter into the nonlinear regime and we have to use Eq. (10) instead of Eq. (11).

2.2. The nonlinear regime

Over the nonlinear regime, we cannot use the decomposition (12) to find uncoupled equations for the different modes. Now, we have to solve the equation for the whole wavepacket. Fortunately, solutions to the 1D sG equation are well known (see, e.g. Rajamaran, 1982). By defining the