New Astronomy 17 (2012) 285–295

Contents lists available at SciVerse ScienceDirect

New Astronomy

journal homepage: www.elsevier.com/locate/newast

The role of flow geometry in influencing the stability criteria for low angular momentum axisymmetric black hole accretion

Sankhasubhra Nag [a], Swagata Acharya [b], Arnab K. Ray [c], Tapas K. Das [d,e,*]

[a] Department of Physics, Sarojini Naidu College for Women, 30, Jessore Road, Kolkata 700028, India
[b] Department of Physics, Ramakrishna Mission Vidyamandira, Belur Math, Kolkata, India
[c] Department of Physics, Jaypee University of Engineering and Technology, A-B Road, Raghogarh 473226, Guna, Madhya Pradesh, India
[d] Harish–Chandra Research Institute, Chhatnag Road, Jhunsi 211019, Allahabad, India
[e] Academia Sinica Institute of Astronomy and Astrophysics, No. 1, Roosevelt Rd., Section 4, Taipei 10617, Taiwan

ARTICLE INFO

Article history:
Received 14 March 2011
Received in revised form 16 June 2011
Accepted 27 June 2011
Available online 7 September 2011

Communciated by M. van der Klis

Keywords:
Accretion
Accretion discs
Black hole physics
Hydrodynamics

ABSTRACT

Using mathematical formalism borrowed from dynamical systems theory, a complete analytical investigation of the critical behaviour of stationary flows in low angular momentum axisymmetric black hole accretion, provides significant insight about the nature of the phase trajectories corresponding to transonic accretion in the steady state, without taking recourse to any explicit numerical method commonly reported in the literature on multi-transonic black hole accretion discs and related astrophysical phenomena. Investigation of an accretion process around a non-rotating black hole, forming different geometrical configurations of the flow structure under the influence of various pseudo-Schwarzschild potentials, reveals that the general profile of the parameter space divisions describing multi-critical accretion, is roughly equivalent for various flow geometries. However, a mere variation of the polytropic index of the flow cannot map a critical solution from one flow geometry to another, since the numerical domain of the parameter space responsible for producing multi-critical accretion does not undergo a continuous transformation in multi-dimensional parameter space. The stationary configuration used to demonstrate the aforementioned findings is shown to be stable under time-dependent linearised perturbations for all kinds of flow geometries, driven by any pseudo-Schwarzschild potential, and using a standard equation of state. Finally, the structure of the acoustic metric corresponding to the propagation of the linear perturbation is discussed for various flow geometries used.

1. Introduction

Astrophysical black holes manifest their presence only gravitationally. No spectral information can be directly obtained from these candidates because of the presence of the event horizon. One can, therefore, rely only on accretion processes to understand their observational signatures (Pringle, 1981; Kato et al., 1998; Frank et al., 2002). At large distances from the accretor, black hole accretion is usually subsonic. The inner boundary condition imposed by the event horizon is determined by the requirement that the flow will be of a supersonic nature very close to the accretor. Black hole accretion, thus, usually exhibits transonic behaviour.

Such physical transonic accretion solutions can be realised mathematically as critical solutions in the phase portraits of the local radial Mach number and the radial distance measured from the event horizon (Ray and Bhattacharjee, 2002; Afshordi and Paczyński, 2003; Ray, 2003a; Ray and Bhattacharjee, 2005b; Ray and Bhattacharjee, 2005a; Chaudhury et al., 2006; Ray and Bhattacharjee, 2006; Ray and Bhattacharjee, 2007a; Bhattacharjee and Ray, 2007; Ray and Bhattacharjee, 2007b; Goswami et al., 2007). To maintain physical transonicity, such critical points will perforce have to be saddle points, which will enable a solution to pass through themselves. In this connection, a "multi-critical" flow refers to the class of accretion configurations which can have more than one critical point accessible to the flow solution. For low angular momentum axisymmetric black hole accretion, it may so happen that the critical features are exhibited more than once in the phase portrait of a stationary solution describing such flows, and the accretion process consequently becomes multi-critical (Liang and Thomson, 1980; Abramowicz and Zurek, 1981; Muchotrzeb and Paczyński, 1982; Muchotrzeb, 1983; Fukue, 1983; Fukue, 1987; Fukue, 2004a; Fukue, 2004b; Lu, 1985; Lu, 1986; Muchotrzeb-Czerny, 1986; Abramowicz and Kato, 1989; Abramowicz and Chakrabarti, 1990; Kafatos and Yang, 1994; Yang and Kafatos, 1995; Caditz and Tsuruta, 1998; Das, 2002; Das et al., 2003; Barai et al., 2004; Abraham et al., 2006; Das et al., 2007; Das and Czerny, 2009).

* Corresponding author at: Harish–Chandra Research Institute, Chhatnag Road, Jhunsi 211019, Allahabad, India.

E-mail addresses: sankhasubhra_nag@yahoo.co.in (S. Nag), acharya.swagata@gmail.com (S. Acharya), arnab.kumar@juet.ac.in (A.K. Ray), tapas@mri.ernet.in (T.K. Das).

1384-1076/$ - see front matter
doi:10.1016/j.newast.2011.06.013

In reality, such weakly rotating sub-Keplerian flows are indeed exhibited in various physical situations, such as detached binary systems fed by accretion from OB stellar winds (Illarionov and Sunyaev, 1975; Liang and Nolan, 1984), semi-detached low-mass non-magnetic binaries (Bisikalo et al., 1998), and super-massive black holes fed by accretion from slowly rotating central stellar clusters (Illarionov, 1988; Ho, 1999) and references therein). Even for a standard Keplerian accretion disc, turbulence may produce such low angular momentum flows (see, e.g., Igumenshchev and Abramowicz (1999), and references therein).

All of the aforementioned multi-critical flow dynamics are important in the astrophysical context. Such multi-critical behaviour allows the formation of standing shocks in low angular momentum axisymmetric black hole accretion (Fukue, 1983; Fukue, 1987; Fukue, 2004a; Fukue, 2004b; Chakrabarti, 1989; Kafatos and Yang, 1994; Yang and Kafatos, 1995; Caditz and Tsuruta, 1998; Fukumura and Tsuruta, 2004; Takahashi et al., 1992; Das, 2002; Das et al., 2003; Abraham et al., 2006; Das et al., 2007; Lu et al., 1997; Lu and Gu, 2004; Nakayama and Fukue, 1989; Nagakura and Yamada, 2008; Nakayama, 1996; Nagakura and Yamada, 2009; Tóth et al., 1998; Das and Czerny, 2009). Standing shocks in rotating astrophysical accretion potentially provide an important and efficient mechanism for the conversion of a significant amount of the gravitational energy into radiation by randomising the directed infalling motion of the accreting fluid. Shocks play an important role in governing the overall dynamical and radiative processes taking place in astrophysical fluid flows around black holes.

Originating at a large distance, subsonic accretion encounters the outermost saddle-type critical point and becomes supersonic. Subjected to an appropriate perturbative environment, such a supersonic flow encounters a shock and becomes subsonic again. The resulting flow has to pass through another saddle-type critical point to meet the inner boundary condition, as imposed by the event horizon. For accretion onto a black hole, the presence of at least two saddle-type critical points is, therefore, a necessary (but not sufficient) condition for shock formation. So multi-critical flow behaviour plays a crucial role in studying the physics of shock formation and related astrophysical phenomena.

To understand the phase-space behaviour of low angular momentum shocked multi-transonic accretion, it is customary to tailor the flow as an autonomous first-order dynamical system, and from it identify the saddle-type critical points of the flow. Next, a global understanding of the flow topologies is obtained upon performing a complete numerical investigation of the nonlinear stationary equations describing the dependence of the velocity on the radial distance. While this is a general practice, through an alternative means reported in the recent literature on accretion (Chaudhury et al., 2006; Mandal et al., 2007; Goswami et al., 2007), it is still possible to semi-quantitatively capture the global behaviour of the transonic solution without resorting to numerical techniques. Equipped with the mathematical formalism of a general dynamical systems approach, these studies have shown that one can derive a clear analytical conception of some of the global features of the flow by analysing the local properties of the critical points.

It is important to note that along with understanding the critical point behaviour of the stationary accretion solution, it is also necessary to ensure that such stationary configurations are stable. This can be accomplished by studying the time evolution of a linear acoustic-like perturbation (around the stationary configuration) in the full time-dependent flow equations. Considering a hydrostatically balanced flow in vertical equilibrium, it has been reported that for accretion onto a non-rotating black hole, driven by various pseudo-Schwarzschild black hole potentials, the characteristic features of the time evolution of the aforementioned perturbation ensure the stability of the stationary configuration (Chaudhury et al., 2006). The result obtained in this way was also shown to be independent of the choice of the black hole potentials used to study accretion flows around a non-rotating black hole.

This study (Chaudhury et al., 2006), however, was carried out for a particular type of flow geometry — hydrostatically balanced flow under vertical equilibrium. Nevertheless, accretion processes onto astrophysical black holes are also studied for two other different flow geometries — flows with constant disc height, and flows under the conical equilibrium (Liang and Thomson, 1980; Abramowicz and Zurek, 1981; Blaes, 1987; Lu et al., 1997; Chakrabarti and Das, 2001; Gu and Foglizzo, 2003) (see Section 2 for further details about these two disc models). These two flow geometries are relatively simple to analyse (in comparison to the flow configuration under vertical equilibrium), without compromising the essential physics involved in multi-transonic black hole accretion phenomena. In addition, these two flows are appropriate to study the low angular momentum inviscid flow configuration as well. Therefore, it will be instructive to investigate the stability of stationary configurations in these flow geometries as well. In other words, one needs to realise if the stationary critical solutions are stable irrespective of the nature of space-time (choice of the black hole potential), as well as the flow geometry (structure of the accretion disc).

This is the precise objective of this work. The stationary and time-dependent low angular momentum axisymmetric accretion around a Schwarzschild black hole, under the influence of a generalised pseudo-Newtonian potential in different flow geometries, has been analysed. The stationary solutions have been considered to investigate their critical point behaviour, and to systematically classify the nature of the critical points which appear in such flows. This is followed by a perturbative study of the full time-dependent flow, in order to track the evolution of the perturbation and make predictions about the stability of the background stationary configuration. Finally, observations have also been made about the nature of the acoustic metric embedded inside the flow.

2. The equations of the flow and its fixed points

When considering a rotating, axisymmetric, inviscid steady flow, the two most pertinent equations are the ones determining the drift in the radial direction (essentially Euler's equation),

$$v\frac{\mathrm{d}v}{\mathrm{d}r}+\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}+\phi'(r)-\frac{\lambda^2}{r^3}=0 \tag{1}$$

and the equation of continuity,

$$\frac{\mathrm{d}}{\mathrm{d}r}(\rho v r H)=0, \tag{2}$$

where $\phi(r)$ is the generalised pseudo-Newtonian potential driving the flow (with the prime denoting a spatial derivative), λ is the conserved angular momentum of the flow, P is the pressure of the flowing gas and $H \equiv H(r)$ is the local thickness of the disc, respectively. The two foregoing equations give the steady continuum distribution of the velocity field, $v(r)$, and the density field, $\rho(r)$. But to close the two equations it will also be necessary to prescribe the functional dependences of both P and H on v and ρ, which, in the steady state regime, will imply an ultimate dependence on r.

Following this requirement, the pressure, P, is first prescribed by an equation of state for the flow (Chandrasekhar, 1939). As a general polytropic it is given as $P = K\rho^{\gamma}$, while for an isothermal flow the pressure is given by $P = \rho\kappa T/\mu m_{\mathrm{H}}$, in all of which, K is a measure of the entropy in the flow, γ is the polytropic exponent, κ is Boltzmann's constant, T is the constant temperature, m_{H} is the mass of a hydrogen atom and μ is the reduced mass, respectively.