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On the tilt of fundamental plane by Clausius' Virial maximum theory

L. Secco *, D. Bindoni

Department of Astronomy, University of Padova, Vicolo Osservatorio, 12, 35100 Padova, Italy

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ABSTRACT

The theory of the Clausius' Virial maximum to explain the fundamental plane (FP) proposed by Secco [Secco, L., 2000. NewA, 5, 403; Secco, L. 2001. NewA, 6, 339; Secco, L. 2005. NewA, 10, 439] is based on the existence of a maximum in the Clausius' Virial (CV) potential energy of a early type galaxy (ETG) stellar component when it is completely embedded inside a dark matter (DM) halo. At the first order approximation the theory was developed by modeling the two-components with two cored power-law density profiles. An higher level of approximation is now taken into account by developing the same theory when the stellar component is modeled by a King-model with a cut-off. Even if the DM halo density remains a cored power-law the inner component is now more realistic for the ETGs. The new formulation allows us to understand more deeply what is the dynamical reason of the FP *tilt* and in general how the CV theory may really be the engine to produce the FP main features. The degeneracy of FP in respect to the initial density perturbation spectrum may be now full understood in a CDM cosmological scenario. A possible way to compare the FPs predicted by the theory with those obtained by observations is also exemplified.

1. On the tilt

It is well known that galaxies of different morphological types cluster around the fundamental plane (FP) (Dressler et al., 1987; Djorgovski and Davis, 1987; Faber et al., 1987; Bender et al., 1992; Djorgovski and Santiago, 1993; Renzini and Ciotti, 1993; Ciotti et al., 1996; Jørgensen, 1999; see, e.g., the review of D'Onofrio et al. (2006), and the references therein) in the three dimensional space of: r_e, effective radius; I_e, the mean effective surface brightness within r_e; σ_o, the central projected velocity dispersion. On the basis of *homology* + *virial theorem* one would expect that the FP equation has to be: $r_e \sim \sigma_o^A I_e^B$ where $A = 2$, $B = -1$. That results completely in disagreement with the observations in different bands. Typical values in B-band are: $A = 1.33 \pm 0.05$; $B = -0.83 \pm 0.03$ (e.g., in D'Onofrio et al. (2006)). These unexpected values produce in the κ coordinate system (Bender et al., 1992) the so called *tilt* that is an increasing of the ratio: dynamical mass M_{dyn} over luminosity L, of this kind:

$$M_{dyn}/L \sim (M_{dyn})^{0.2} \tag{1}$$

Many attempts have been done in order to understand the FP *tilt* which is also one of the common features either for galaxy FPs or for the FPs of all virialized structures which all together define the so called *cosmic meta-plane* (Burstein et al., 1997). The review of D'Onofrio et al. (2006) may help the reader to take into account the more recent efforts to solve the hard problem of finding an explanation of the trend (1) when the *K-band* is also considered and then the population effect has to be ruled out. Actually it is possible to explain the trend observed in the B-band as a metallicity sequence of an old stellar population (Maraston, 1999). However the M_{dyn}/L values in the K-band are independent of metallicity even if the tilt is observed (Pahre et al., 1998). A secondary effect is then needed to explain the K-band tilt (Gerhard et al., 2001).

The Clausius' Virial theory (TCV) of FP has the aim to propose a dynamical mechanism able to produce the required effect on a huge range of mass scales from globular clusters to galaxy clusters. The purpose is to prove that it may be possible to change A, B exponents (from the expected values 2, −1) without breaking *homology* + *virial equilibrium*. It is based on the existence of a special virial configuration characterized by a maximum in the Clausius' Virial potential energy (CV) which, on galaxy mass scale, refers to a baryonic (stellar, B) component when it is completely embedded inside a DM halo (D component). At the first order approximation (linear) the two-components are modeled with two power-law density profiles and two infinitesimal cores. The general strategy is described in many papers (Secco, 2000, 2001, hereafter LS1, Marmo and Secco, 2003; Secco, 2005, hereafter LS5).

Now we move from a linear approach of TCV to a non-linear one that is to an higher level of approximation in which the stellar component is built up by a King-model with a cut-off. Even if the DM halo density remains a power-law the inner component is actually more realistic for the ETGs. The new formulation allows us to understand more deeply the physical reasons which produce

* Corresponding author. Tel.: +39 49 8278236; fax: +39 49 8278212.
E-mail address: luigi.secco@unipd.it (L. Secco).

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the FP *tilt* and the role of the main involved quantities, particularly that of I_e. Moreover we may begin the comparison between the expected edge-on FPs with those obtained by observations (e.g. that of Djorgovski and Davies (1987)) and try to reproduce in κ-space the *tilt* fit-equation of Burstein et al. (1997). Its theoretical derivation may explain why the FP is degenerate in respect to the initial density perturbation spectrum in a CDM scenario as already underlined by Djorgovski (1992). Some initial examples of theoretical FP calibration will be given in Sections 7 and 9, for some special choice of theoretical parameters. A more complete discussion is still in progress.

2. General strategy of TCV

Briefly summarizing, the general strategy consists to use the two-component tensor virial theorem (e.g. Brosche et al., 1983; Caimmi and Secco, 1992) to describe the virial configuration of the baryonic component embedded in a DM halo at the end of relaxation phase (see Bindoni and Secco, 2008). It reads:

$$2(T_u)_{ij} = (V_u)_{ij} \quad (u = B, D;\ \ i,j = x,y,z) \tag{2}$$

According to the scalar virial for one component, the potential energy tensor, which has to enter into the tensor virial equations, is the Clausius' Virial tensor, $(V_u)_{ij}$, built-up of the *self-potential energy tensor*, $(\Omega_u)_{ij}$, and the *tidal potential energy tensor*, $(V_{uv})_{ij}$. Then, according to the scalar virial theorem, the trace of CV tensor, in the case of stellar component, has to be read:

$$\begin{aligned} V_B &= \Omega_B + V_{BD} \\ \Omega_B &= \int \rho_B \sum_{r=1}^{3} x_r \frac{\partial \Phi_B}{\partial x_r}\, d\vec{x}_B = \int \rho_B(\vec{r}_B \cdot \vec{f}_B)\, d\vec{x}_B \\ (V_{BD}) &= \int \rho_B \sum_{r=1}^{3} x_r \frac{\partial \Phi_D}{\partial x_r}\, d\vec{x}_B = \int \rho_B(\vec{r}_B \cdot \vec{f}_D)\, d\vec{x}_B \end{aligned} \tag{3}$$

where ρ_B is the B component density and $\vec{f}_B$, $\vec{f}_D$ are the force per unit mass due to the self and DM gravity, respectively, at the point $\vec{r}_B$ and Φ_B, Φ_D are the respective potentials.

Conversely, the total potential energy tensor of the B component is: $(\Omega_B)_{ij} + (W_{BD})_{ij}$, where the interaction energy tensor is: $(W_{BD})_{ij} = -\frac{1}{2}\int \rho_B(\Phi_D)_{ij}\, d\vec{x}_B$; and the potential tensor due to the DM (e.g., Chandrasekhar, 1969) is: $(\Phi_D)_{ij} = G\int \rho_D(\vec{x}')^3 \frac{(x_i - x_i')(x_j - x_j')}{|\vec{x} - \vec{x}'|^3}\, d\vec{x}_D$.

To be noted that in general: $(V_{BD})_{ij} \neq (W_{BD})_{ij}$, the difference gives the residual energy tensor (Caimmi and Secco, 1992).

We will describe a re-formulation of TCV in the case in which the two-component model is built up of: a bright B stellar component with a King (1962) truncated density profile completely embedded in a DM frozen halo, D, with a cored power-law mass density distribution.

3. Why introducing King's model

3.1. End of relaxation phase

The violent relaxation mechanism leads to an equipartition of energy per unit mass and not per particles (see, e.g., the review of Bindoni and Secco (2008), and references therein). If σ is the velocity dispersion, assumed to be the same for every star mass, integration of the distribution function, $f(E)$, over the velocities (Binney and Tremaine, 1987, Chapter 4; Combes et al., 1995, Chapter 4), yields the density:

$$\rho(r) = \rho_1 e^{-U(r)/\sigma^2} \tag{4}$$

where the total energy per unit mass is: $E = (1/2)v^2 + U$; (v and U are velocity and potential energy per unit mass, respectively). On the other hand, Poisson equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dU}{dr}\right) = 4\pi G \int f(E)\, d\vec{v} \tag{5}$$

becomes by means of Eq. (4):

$$\frac{d}{dr}\left(r^2 \frac{d\ln\rho}{dr}\right) = -\frac{4\pi G}{\sigma^2} r^2 \rho \tag{6}$$

with the solution:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2} \tag{7}$$

In turn, Eq. (4) gives:

$$2\ln\left(\frac{3}{\sqrt{2}}\frac{r}{r_c}\right) = U(r)/\sigma^2 \tag{8}$$

when a core radius $r_c = 3\sigma(4\pi G\rho_o)^{-1/2}$ is introduced in order to avoid an infinite value of the central density ρ_o. r_c corresponds to the radius at which the projected density of the isothermal sphere falls to roughly half of its central value. Eq. (8) gives us the asymptotic behavior as soon as r is greater of about $2r_c$:

$$U(r) \approx 2\sigma^2 \ln(r/r_c) \tag{9}$$

which means again from Eq. (4), an isothermal behavior, $\rho(r) \propto r^{-2}$ as $r \to \infty$.

3.2. Problems with isothermals

The isothermal energy distribution function extends spatially to infinity with infinite mass and so does not be suitable to represent a real elliptical galaxy.

Since 1965, Ogorodnikov has highlighted that: in order to find the most probable phase distribution function for a stellar system in a stationary state, the phase volume has to be truncated in both coordinate and velocity space. While in the velocity space the truncation arises spontaneously due to the existence of escape velocity, the introduction of a cut-off in the coordinate space appears, on one side, necessary in order to obtain a finite mass M and radius R, but, on the other, very problematic.

A similar difficulty also appears on the thermodynamical side, for which an extensive literature exists (from: Lynden-Bell and Wood, 1968; Horowitz and Katz, 1978; White and Narayan, 1987, until, e.g., Bertin and Trenti, 2003, and references therein). By using the standard Boltzmann–Gibbs entropy:

$$S = -\int f \ln f\ d^3x d^3v \tag{10}$$

defined by the *distribution function*, $f(\vec{x}, \vec{v})$ (hereafter *DF*), in the μ phase-space, and looking for what maximizes the entropy of the same stellar system, the conclusion is: the *DF* which plays this role in (10) is that of the isothermal sphere. But, the maximization of **S**, subject to fixed mass M and energy E, leads again to a *DF* that is incompatible with finite M and E (see, e.g., Binney and Tremaine, 1987, Chapter 4; Merritt, 1999; Lima Neto et al., 1999; Marquez et al., 2001, and references therein).

Our limited contribution to the wide discussion existing in the literature will be to underline as in a stellar component, embedded in a second dark matter subsystem (e.g., Ciotti, 1999, and references therein), a truncation is spontaneously introduced in coordinate space, due to the presence of a scale length induced from the dark halo, as long as virial equilibrium holds. That is the *tidal radius*