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Gravitational lensing: From micro to nano

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ABSTRACT

Different regimes of gravitational lensing depend on lens masses and roughly correspond to angular distance between images. If a gravitational lens has a typical stellar mass, this regime is named a microlensing because a typical angular distance between images is about microarcseconds in the case when sources and lenses are located at cosmological distances. An angular distance depends on a lens mass as a square root and therefore, if a lens has a typical Earth-like planet mass of $10^{-6}M_{\odot}$, such a regime is called nanolensing. Thus, generally speaking, one can call a regime with a planet mass lens a nanolensing (independently on lens and source locations). So, one can name searches for planets with gravitational lens method a gravitational nanolensing. There are different methods for finding exoplanets such as radial spectral shifts, astrometrical measurements, transits, pulsar timing etc. Gravitational microlensing (including pixel-lensing) is among the most promising techniques if we are interested to find Earth-like planets at distances about a few astronomical units from the host star.

1. Gravitational lensing: introduction

Gravitational lensing is based on a simple physical phenomenon that light trajectories are bent in a gravitational field (in some sense a gravitating body attracts photons). For the first time this fact was discussed by Newton (1704), but the first derivation of the light bending angle in the framework of Newtonian gravity was published by Soldner (1804). Using a weak gravitational field approximation in general relativity (GR) the correct bending angle is described by the following expression obtained by Einstein (1916) just after his formulation of GR

$$\delta\varphi = \frac{4GM}{c^2 p}, \tag{1}$$

where M is a gravitating body mass, p is an impact parameter, c is a speed of light, G is the Newton constant. If $M = M_{\odot}$ and $p = R_{\odot}$ are solar mass and radius, respectively, the angle is equal to $1.''75$. In 1919 this law was firstly confirmed for observations of light ray bending by the Solar gravitational field near its surface (Dyson et al., 1920). Therefore, the Einstein prediction about light bending was confirmed by observations very soon after its appearance.

Using Eq. (1) one can introduce the gravitational lens equation

$$\vec{\eta} = D_s\vec{\xi}/D_d - D_{ds}\vec{\Theta}(\vec{\xi}), \tag{2}$$

where D_s is a distance between a source and observer, D_d is a distance between a gravitational lens and observer, D_{ds} is a distance between a source and a lens, $\vec{\eta}, \vec{\xi}$ define coordinates in source and lens planes, respectively, and

$$\vec{\Theta}(\vec{\xi}) = 4GM\vec{\xi}/c^2\xi^2. \tag{3}$$

Taking that the right hand side of Eq. (2) is zero ($\vec{\eta} = 0$) and substituting $\vec{\Theta}$ from Eq. (3), we obtain the so-called Einstein–Chwolson radius[1] (Schneider et al., 1992): $\xi_0 = \sqrt{4GMD_dD_{ds}/(c^2D_s)}$ and the Einstein–Chwolson angle: $\theta_0 = \xi_0/D_d$. If $D_s \gg D_d$, we have

$$\theta_0 \approx 2'' \times 10^{-3}\left(\frac{GM}{M_{\odot}}\right)^{1/2}\left(\frac{\text{kpc}}{D_d}\right)^{-1/2}. \tag{4}$$

[1] Chwolson described circular images (Chwolson, 1924) and Einstein obtained basic expressions for gravitational lensing (Einstein, 1936). Moreover, it was found that Einstein analyzed gravitational lensing phenomenon in his unpublished notes in 1912 (Renn et al., 1997).

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1.1. Regimes of gravitational lensing

There is a number of reviews and books on gravitational lensing (Schneider et al., 1992; Wambsganss, 1993; Refsdal and Surdej, 1994; Zakharov, 1997b; Roulet and Mollerach, 2002; Claeskens and Surdej, 2002). Gravitational lensing in the strong gravitational field approximation was also analyzed (Frittelli et al., 2000; Bozza et al., 2001; Virbhadra and Ellis, 2002; Virbhadra and Keeton, 2008; Virbhadra, 2009).

As it is shown below, in the framework of the simplest point-like lens model (the Schwarzschild lens) angular distances between images are about $2\theta_0$ and the angle is proportional to the square root of the lens mass for fixed other parameters (including distances). So, if a gravitational lens has a typical galactic mass of about $10^{12}\,M_\odot$, a typical distance between images is about a few angular seconds (it corresponds to the standard gravitational macro-lensing regime); if a gravitational lens has a typical stellar mass of about $M_\odot$, a typical distance between images is about 10^{-6} arcsecond (it corresponds to the gravitational microlensing regime); if a gravitational lens has a typical Earth-like planet mass of about $10^{-6}M_\odot$, a typical distance between images is about 10^{-9} arcsecond (it corresponds to the gravitational nanolensing regime). Really, 10^{-9} arcsecond is very small angle and to imagine it one can try to take a look at one inch coin from the distance of about 4.5×10^9 km (or about 30 AU), which is roughly equal to the distance between Sun and Neptune.

Naturally, at the moment there is no way to resolve micro- and nano- images but there is a way to discover photometrical features of the phenomena by monitoring light curves of background sources (Byalko, 1970). Moreover, there are projects planning to reach angular resolutions at a microarcsecond level (in different spectral bands) such as NASA Space Interferometry Mission (SIM), ESA Global Astrometric Interferometer for Astrophysics (Gaia) (Lindegren and Perryman, 2000), NASA MicroArcsecond X-Ray Imaging Mission (MAXIM) (Cash et al., 2000; White, 2000), Russian RadioAstron. It is planned to reach even a nanoarcsecond level in mm band with space–ground interferometry technique with Millimetron mission.[2]

If a gravitational lens is one of the closest galaxies at a distance $D_d = 100$ kpc with mass $M = 10^{12}\,M_\odot$, we have $\theta_0 \approx 200''$. If a gravitational lens is a star in our Galaxy at a distance 1 kpc, we have $M = M_\odot$, and $\theta_0 \approx 2''\times10^{-3}$ (similar, if a lens is a planet at the same distance with a mass about $M = 10^{-6}M_\odot$ then $\theta_0 \approx 2''\times10^{-6}$). According to a standard terminology proposed many years ago, if a lens mass is about $M_\odot$ ($M = 10^{-6}\,M_\odot$) we call this lensing regime a microlensing (nanolensing) independently on locations of sources and lenses. More generally speaking, searches for planets through their impacts on gravitational lensing may be named a gravitational nanolensing.

We could introduce dimensionless variables

$$\vec{x} = \vec{\xi}/\xi_0, \quad \vec{y} = D_s\vec{\eta}/(\xi_0 D_d), \quad \vec{\alpha} = \vec{\Theta}D_{ds}D_d/(D_s\xi_0), \tag{5}$$

then we have gravitational lens equation in the dimensionless form:

$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x}) \quad \text{or} \quad \vec{y} = \vec{x} - \vec{x}/x^2. \tag{6}$$

The gravitational lens effect may lead to a formation of several images instead of one (see, for instance, Schneider et al., 1992; Zakharov, 1997b). We have two images (or one ring) for the Schwarzschild point lens model, as one can see in Fig. 1. The total area of the two images is larger than a source area. The ratio of a sum of these two image areas and a source area is called gravitational lens amplification A and it is a result of gravitational focusing. For example, if a circular source with a radius r and its area πr^2 is located near a position of gravitational lens on a celestial sphere then an area of ring image is equal to $2\pi r R_{EC}$ (the width of the ring is r and its circumference is $2\pi R_{EC}$, since we express all distances in Einstein–Chwolson radii – R_{EC}) and, therefore, magnification is $2R_{EC}/r$ (thus one could calculate an asymptote for a magnification in a limit $r \to 0$ by the geometrical way). That is a reason to call gravitational lensing as gravitational focusing. As one can see the angular distance between two images is about angular size of so-called the Einstein–Chwolson cone with the angle $2\theta_0$ (it corresponds to the Einstein–Chwolson diameter).

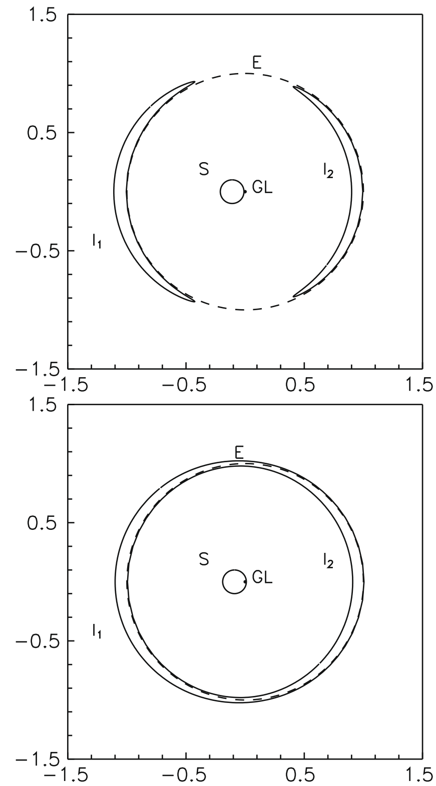

Fig. 1. Image formation for a circular source S with a radius $r = 0.1$ and for two different distances d between a source center and gravitational lens GL on the celestial sphere: $d = 0.11$ (top panel) and $d = 0.09$ (bottom panel), where I_1 and I_2 are images and E is the Einstein–Chwolson ring.

2. Gravitational microlensing

There is a number of reviews on gravitational lensing (Wu, 1994; Paczynski, 1996; Roulet and Mollerach, 1997, 2002; Zakharov and Sazhin, 1998; Mao, 1999; Jetzer, 1999; Zakharov, 2003, 2005, 2008b; Mao, 2008). If a source S lies on the boundary of the Einstein–Chwolson cone, then we have $A = 1.34$. The microlensing time is defined typically as a half of the total time of crossing the cone T_0:

$$T_0 = 3.5\ \text{months}\ \cdot\sqrt{\frac{M}{M_\odot}\frac{D_d}{10\ \text{kpc}}}\cdot\frac{300\ \text{km/s}}{V},$$

where V is the perpendicular component of a velocity of a dark body. If we suppose that the perpendicular component of a velocity of a dark body is equal to ~300 km/s (that is a typical stellar velocity in Galaxy), then a typical time of crossing Einstein cone is about 3.5 months. Thus, a luminosity of a source S is changed with that time. We will give numerical estimates for parameters of the microlensing effect. If the distance between a dark body and the Sun is

[2] See, http://www.asc.rssi.ru.