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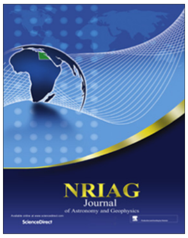

FULL LENGTH ARTICLE

Execution of novel explicit RKARMS(4,4) technique in determining initial configurations of extra-solar protoplanets formed by disk instability

Gour Chandra Paul [a,*], **Sukumar Senthilkumar** [b,c,d]

[a] *Department of Mathematics, University of Rajshahi, Rajshahi 6205, Bangladesh*
[b] *Universiti Sains Malaysia, School of Mathematical Sciences, Pulau Pinang 11800, Malaysia*
[c] *Center for Advanced Image and Information Technology, Division of Computer Science and Engineering, School of Electronics & Information Engineering, Chon Buk National University, 664-14, 1Ga, Deok Jin-Dong, Jeonju, Chon Buk 561-756, South Korea*
[d] *School of Computing Science and Engineering, Vellore Institute of Technology University, Tamilnadu, Vellore 632014, India*

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Abstract Implementation of a novel embedded Runge–Kutta fourth order four stage arithmetic root mean square technique to determine initial configurations of extra-solar protoplanets formed by gravitational instability is the main goal of this present paper. A general mathematical framework for the introduced numerical technique is described in addition to error estimation description. It is noticed that the numerical outputs through the employed novel RKARMS(4,4) method are found to be more effective and efficient in comparison with the results obtained by the classical Runge–Kutta technique.

1. Introduction

From literature review it is noticed that the ever-increasing advances in high performance computer technology have enabled several researchers towards science and engineering to employ novel numerical techniques to simulate physical phenomena. Intensive techniques are frequently required for the solution of real time practical problems and they often need the systematic application of a range of elementary techniques. In the development of new numerical methods, simplifications required to be made to progress towards an optimal solution. As a result, numerical algorithms do not usually give the exact answer to a given problem, or they can only tend towards a solution getting closer and closer with each iteration. Numerical techniques exhibit certain computational characteristics during their real time implementation. It is significant to consider these characteristics while selecting a specific technique for implementation. The characteristics which are critical to the success of implementation are accuracy, rate of convergence, numerical stability and efficiency. Numerical

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[*] Corresponding author. Tel.: +88 01712187595; fax: +88 0721 750064.
E-mail addresses: pcgour2001@yahoo.com (G.C. Paul), ssenthilkumar1974@yahoo.co.in (S. Senthilkumar).

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algorithms must review the factors such as, determination of the correctness of various steps, reduction of the number of steps, if necessary, and increase in the speed of solving the problem, respectively.

To solve many different problems under signal processing, communication, electronic and transistor circuits, Runge–Kutta (RK) method is being applied to obtain the required solution (Alexander and Coyle, 1990). Shampine and Gordon (1975) discussed the normal order of a RK algorithm having the approximate number of leading terms of an infinite Taylor series, which calculates the trajectory of a moving point. Yaakub and Evans (1993) presented a new fourth order RK method based on the root mean formula for solving initial value problems (IVPs) in numerical studies. A new embedded fourth order RK method which is actually two different RK methods but of the same order $p = 4$ has been introduced by Evans and Yaakub (1995). Bader (1987, 1998) introduced the RK–Butcher algorithm for finding the truncation error estimates, intrinsic accuracies and the early detection of stiffness in coupled differential equations arising in theoretical chemistry problems. Yaakub and Evans (1999) introduced a new fourth order RK technique for IVPs with error control. Butcher (1987, 1990, 2003) derived the best RK pair along with an error estimates and by all statistical measures it appeared as RK–Butcher algorithms. In order to overcome step-size constraint imposed by numerical stability, many new techniques have been developed in recent past. To confirm this, recently Ponalagusamy and Senthilkumar (2009) proposed a novel fourth order embedded RKARMS(4,4) technique based on RK arithmetic mean and root mean square with error control in detail to solve the real time application problems efficiently in image processing under CNN model. A detailed illustration related to the local truncation error (LTE), the global truncation error (GTE), error estimates and control for fourth order and four stage RK numerical algorithms is eventually addressed by Senthilkumar (2009).

The formation of planetary systems has been a topic of interest to the mankind ever since the dawn of civilization. However, scientific theories for the formation of the system largely date from Descartes (1644) when he proposed his vortex theory in this regard. Since that time many theories have been advanced. In most cases these theories were primarily speculative because of the lack of observational characteristics of the system. Fortunately, for the theorists of today, there are some convenient observational constrains of the system. The two end mechanisms, namely core accretion and disk instability, in principle, can form gas giant protoplanets. Though the core accretion mechanism has been, so far, adopted as the main theory of planetary formation both in our solar system and elsewhere, it fails to explain properly the recently discovered extrasolar protoplanets by direct imaging (see Dodson-Robinson et al., 2009). With this difficulty encountered by the core accretion models, the disk instability model, once in vague, has been reformulated with fragmentation from massive protoplanetary disks and has been advanced through the investigations of many authors (e.g., Boss, 1997; Mayer et al., 2002, 2004; Boley et al., 2010; Cha and Nayakshin, 2011). But this model is also criticized by some investigations with the argument that disk instabilities are unable to lead to the formation of self-gravitating dense clumps (Pickett et al., 2000; Cai et al., 2006; Boley et al., 2007). Although some questions arise as to whether stable protoplanets could be formed or not by disk instability, the idea is believed to be a promising route to the rapid formation of giant planets in our solar system and elsewhere (Boss, 2007). Unfortunately, the initial structures of the protoplanets formed via gravitational instability are still unknown and different numerical models can be found to report different configurations (Helled and Schubert, 2008; Helled and Bodenheimer, 2011).

It is pertinent to point out here that depending upon opacities of grains, present in protoplanets, as well as on initial conditions different investigators assumed different heat transports at different regions of protoplanets at different stages of their evolution. DeCampli and Cameron (1979), in their investigation, assumed initial protoplanets to be fully convective with a thin outer radiative zone as a consequence of higher opacity and much work has since then been devoted to the evolution of planetary system including our own from such types of initial protoplanets (e.g., Bodenheimer et al., 1980; Wuchterl et al., 2000; Helled et al., 2008; Helled and Bodenheimer, 2011). It is well-known that depending on obeying the law $L/4\pi R^2 = (\text{surface opacity})^{-1}$, where L represents the luminosity and R is the protoplanetary radius, or on slow contraction, initial protoplanets may be fully convective (see DeCampli and Cameron, 1979), which is consistent with Helled et al. (2005). To investigate planetary evolution from such types of protoplanets, a series of studies were conducted by Paul et al. (2008, 2012, 2013) and the obtained results were found to be in good agreement with the estimates by other investigations (see e.g., Helled and Schubert, 2008; Helled et al., 2008). However, recently Boss (1998, 2002, 2007) in his investigations assumed the protoplanets to be in radiative equilibrium, which is consistent with earlier investigation by Bodenheimer (1974) who calculated a completely radiative Jovian mass structure assuming a constant grain opacity of $0.14\,\text{cm}^2\,\text{g}^{-1}$. It is of interest to note here that in the case of radiative heat transfer, conduction is also taken part (Böhm-Vitense, 1997). Based on the idea, Paul et al. (2008) investigated initial structure of a Jovian mass protoplanet, which was further extended by Paul and Bhattacharjee (2013) for investigating initial structures of extra solar protoplanets and the obtained results were found to be consistent with the results reported in some studies with rigorous treatment of the problem (see Paul et al., 2013).

In this communication we intend to reinvestigate the model of Paul and Bhattacharjee (2013) assuming heat transport following them to be conductive-radiative by a novel explicit RKARMS(4,4) method in order to test its validity and efficiency and to see how our computed results compare the estimates obtained with other investigations.

The rest of the article is structured as follows. The theoretical foundation of the problem in addition to boundary conditions is presented in Section 2. Numerical technique is adopted in Section 3. A brief description of the explicit RKARMS(4,4) technique along with local truncation error and error control is addressed in Section 4 and in its subsection. In Section 5, discussion of the obtained results as well as conclusion is presented.

2. Theoretical foundation

As in Paul et al. (2008) and Paul and Bhattacharjee (2013), the structure of a protoplanet, assuming the heat transport to be conductive-radiative, is given by the following set of equations: