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A relativistic signature in large-scale structure

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ABSTRACT

In General Relativity, the constraint equation relating metric and density perturbations is inherently nonlinear, leading to an effective non-Gaussianity in the dark matter density field on large scales—even if the primordial metric perturbation is Gaussian. Intrinsic non-Gaussianity in the large-scale dark matter overdensity in GR is real and physical. However, the variance smoothed on a local physical scale is not correlated with the large-scale curvature perturbation, so that there is no relativistic signature in the galaxy bias when using the simplest model of bias. It is an open question whether the observable mass proxies such as luminosity or weak lensing correspond directly to the physical mass in the simple halo bias model. If not, there may be observables that encode this relativistic signature.

1. Introduction

In Newtonian gravity, the Poisson equation is a linear relation between the gravitational potential and the matter overdensity. By contrast, in General Relativity (GR) this is replaced by a nonlinear relation, which introduces mode coupling between large and small scales [1–3]. The original result has been confirmed by a number of independent calculations [4–9]. Similar mode coupling can be produced in Newtonian gravity by local-type primordial non-Gaussianity of the gravitational potential [10,11]. The GR effect has therefore previously been interpreted as an effective local non-Gaussianity on very large scales [6,12–15].

Recently two papers have argued that a "separate universe" approach can be used to show that *no* scale-dependent bias arises from the GR corrections on large scales [16,17]. The argument is that the nonlinear coupling between long-wavelength perturbations on a scale λ_L, and the small-scale variance, $\sigma_S^2 = \langle\delta_S^2\rangle$, on a scale λ_S, vanishes under a local coordinate rescaling and hence is unobservable.

The separate universe approach [18,19] has proved to be a powerful tool to understand the origin of large-scale structure, and primordial non-Gaussianity, from inflation. Accelerated expansion in the very early Universe stretches initial small-scale vacuum fluctuations up to scales much larger than the Hubble scale at the end of inflation. Spatial gradients for such long-wavelength modes become small relative to the local Hubble time, and for many scales of interest, the perturbed universe can be treated as a patchwork of "separate universes", each locally obeying the classical Friedmann–Lemaître–Robertson–Walker (FLRW) evolution of an unperturbed universe.

The separate universe approach is particularly useful for studying nonlinear perturbations on large scales [18,20]. For

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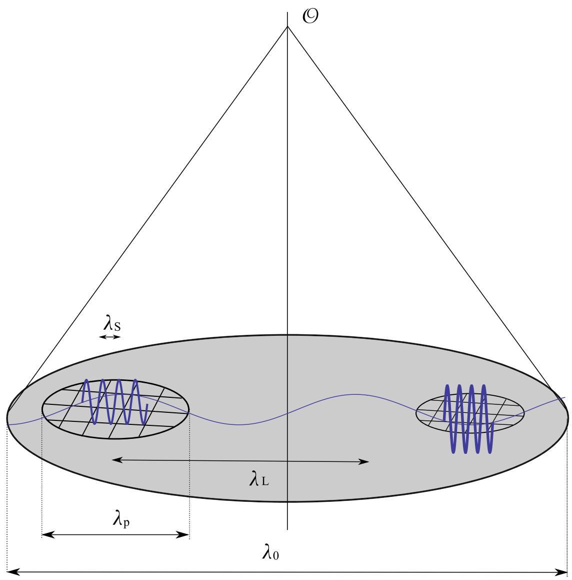

Fig. 1. Schematic of the various scales in (2).

adiabatic perturbations, each separate universe patch follows locally the same evolution as the unperturbed "background" cosmology. The only difference between separate patches is the local expansion, characterised by the comoving metric perturbation ζ. This is defined to be the local perturbation of the integrated expansion rate with respect to a background flat reference cosmology, $\delta N = N - \bar{N}$, where $N = \int dt\, \Theta/3$.

An important consequence of the uniqueness of the local evolution for adiabatic perturbations is that ζ is conserved on large scales where the separate universe approach is valid [19,21,22].

In each patch, the comoving spatial line element is (see [17])

$$ds^2_{(3)} = e^{2\zeta}\,\delta_{ij} dx^i dx^j\,. \tag{1}$$

There is a global *background* which must be defined with respect to some scale λ_0, at least as large as all the other scales of interest, i.e., at least as large as our presently observable Universe. It is important to distinguish this from the scale of the separate universe patches, λ_P. This is large enough for each patch to be treated as locally homogeneous and isotropic, but patches must be stitched together to describe the long-wavelength perturbations on a scale $\lambda_L \gg \lambda_P$. Thus, following [19], we require a hierarchy of scales (see Fig. 1):

$$\lambda_0 > \lambda_L \gg \lambda_P \gg \lambda_S\,. \tag{2}$$

The local observer in a separate universe patch cannot observe the effect of ζ_L, which is locally homogeneous on the patch scale λ_P. However, local coordinates can be defined only locally and the long mode curvature perturbation is observable through a mapping from local to global coordinates.

2. The physical effect of curvature within the observable universe

In Newtonian gravity the only constraint on initial conditions is the Poisson equation, which provides a linear relation between the overdensity and the gravitational potential at all orders

$$\nabla^2 \Phi_N = -\frac{3}{2} a^2 H^2 \delta\,. \tag{3}$$

Thus if the initial Newtonian potential Φ_N is Gaussian, then so is the initial density field δ. In GR, the nonlinear energy constraint equation for irrotational dust is [23]

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + R^{(3)} = 16\pi G\rho + 2\Lambda\,, \tag{4}$$

where ρ is the comoving matter density, Λ is the cosmological constant, $\Theta = \nabla_\mu u^\mu$ is the expansion rate of the matter 4-velocity, σ is its shear, and $R^{(3)}$ is the Ricci curvature scalar of the 3-dimensional space orthogonal to u^μ. At first order in perturbations about an FLRW cosmology, the energy constraint combines with the momentum constraint to give the relativistic version of the Poisson equation (3), where Φ_N is replaced by Φ, i.e. the spatial metric perturbation in longitudinal gauge, and δ is the synchronous comoving gauge density contrast. Note that $\Phi = 3\zeta/5$ at first order. At second order, at the start of the matter era, using the relation between $R^{(3)}$ and ζ, we obtain [6]

$$\nabla^2\zeta - 2\zeta\nabla^2\zeta + \frac{1}{2}(\nabla\zeta)^2 = -\frac{5}{2}a^2H^2\delta\,. \tag{5}$$

Consider a Gaussian distribution of ζ. We separate ζ and δ into independent long- and short-wavelength modes, $\zeta = \zeta_L + \zeta_S$ and $\delta = \delta_L + \delta_S$, where the wavelength of the long modes λ_L obeys (2); in particular, $\lambda_L \gg \lambda_P$. To leading order in ζ_S and ζ_L, and neglecting gradients of ζ_L relative to those of ζ_S, the initial constraint (5) implies $\nabla^2\zeta_L = -5a^2H^2\delta_L/2$ and

$$\nabla^2\zeta_S - 2\zeta_L\nabla^2\zeta_S = -\frac{5}{2}a^2H^2\delta_S. \tag{6}$$

The second term on the left represents the long–short mode coupling.

Within a local patch on a scale λ_P, it is possible to redefine the background spatial coordinates to absorb the effects of the long-wavelength perturbations ζ_L, following [17]:

$$\tilde{x}^i = x^i + \xi^i, \quad \xi^i = \zeta_L x^i. \tag{7}$$

If we neglect gradients of the long mode, this transformation eliminates ζ_L from the spatial metric (1)

$$ds^2_{(3)} = e^{2\zeta_L} e^{2\zeta_S}\delta_{ij}dx^i dx^j = e^{2\zeta_S}\delta_{ij} d\tilde{x}^i d\tilde{x}^j. \tag{8}$$

This transformation holds for each *single* patch (see Fig. 1).

Since this is a purely spatial coordinate transformation, the curvature and density perturbations transform as scalars,

$$\tilde{\zeta}_S(\tilde{x}) = \zeta_S(x), \qquad \tilde{\delta}_S(\tilde{x}) = \delta_S(x), \quad \text{where } \tilde{x} = \left[1 + \zeta_L(x)\right]x\,. \tag{9}$$

The constraint equation (6) becomes

$$\tilde{\nabla}^2\tilde{\zeta}_S(\tilde{x}) = -\frac{5}{2}a^2H^2\tilde{\delta}_S(\tilde{x}). \tag{10}$$

Thus in the new local coordinates, in one patch of size λ_P, we have a linear Poisson equation and the long–short mode coupling appears to be absent. This confirms the fact that the local observer in a separate universe patch cannot observe the effect of the locally homogeneous perturbation ζ_L, as argued in [16,17].

The original coordinates x^i define a global chart, which is essential for defining random fields such as ζ_L on large scales, and the long mode curvature perturbation enters through the mapping from local to global coordinates. Indeed, a coordinate transformation that depends on a random field is not a new concept in large-scale structure. The situation here is reminiscent of the redshift-space distortion map, where the random field is given by the peculiar velocities (which are in turn generated by large-scale density perturbations). Because of the nonlinear nature of this map, an initially Gaussian field in real space becomes non-Gaussian in redshift space [24,25].

As shown in [17], $\tilde{\zeta}(\tilde{x})$ and $\tilde{\delta}(\tilde{x})$ are Gaussian fields with respect to the local coordinates $\tilde{x}^i$. By (9), $\tilde{\delta}(\tilde{x}) = \delta(x)$ and so we recover a non-Gaussian distribution for the density field with respect to the global coordinates, x^i. See Fig. 2 for a schematic illustration of this.