Physics of the Dark Universe 13 (2016) 87–91

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Effects of quantum fluctuations of metric on the universe

Rongjia Yang *

College of Physical Science and Technology, Hebei University, Baoding 071002, China
Hebei Key Lab of Optic-Electronic Information and Materials, China
State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 1 February 2016
Received in revised form
20 April 2016
Accepted 26 April 2016

Keywords:
Modified gravity
Inflation
Dark energy
Cosmology

ABSTRACT

We consider a model of modified gravity from the nonperturbative quantization of a metric. We obtain the modified gravitational field equations and the modified conservational equations. We apply it to the FLRW spacetime and find that due to the quantum fluctuations a bounce universe can be obtained and a decelerated expansion can also possibly be obtained in a dark energy dominated epoch. We also discuss the effects of quantum fluctuations on inflation parameters (such as slow-roll parameters, spectral index, and the spectrum of the primordial curvature perturbation) and find values of parameters in the comparing the predictions of inflation can also work to drive the current epoch of acceleration. We obtain the constraints on the parameter of the theory from the observation of the big bang nucleosynthesis.

1. Introduction

A great number of astronomical observations have confirmed that the Universe is experiencing an accelerated expansion. In the framework of general relativity an unknown energy component, dubbed as dark energy, is usually introduced to explain this phenomenon. The simplest candidate of dark energy is the vacuum energy with a constant equation of state (EoS) parameter $w = -1$. This model is consistent with most of the current astronomical observations, but it suffers from the cosmological constant problem [1] and age problem [2] as well. Therefore it is natural to consider more complicated cases. The most popular attempt is to propose modifications of the Einstein–Hilbert Lagrangian by adopting different functions of the Ricci scalar, known as $f(R)$ theories, which have been studied extensively [3–7]. However, the fourth order field equations in $f(R)$ theories make it is hard to analyze. Analogous to $f(R)$ theories, a new scenario based on the modification of the teleparallel gravity, called $f(T)$ theory, was proposed to explain the accelerated expansion of the Universe [8,9]. Recently, it has been shown that one can obtain modified gravity from Heisenberg's nonperturbative quantization [10–12]. According to this technique, the classical fields appearing in the corresponding field equation are replaced by operators of the fields. Think of general relativity, we have the operator Einstein equations

$$\hat{G}_{\mu\nu} \equiv \hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = k^2\hat{T}_{\mu\nu}, \tag{1}$$

where $k^2 = 8\pi G$ and we take $c = 1$. All geometric operators, such as $\hat{\Gamma}^a_{bc}$, $\hat{R}^a{}_{bcd}$, and $\hat{R}_{ab}$, are defined in the same way as in the classical case by replacing the classical quantities with the corresponding operators [10]. Heisenberg's technique offers one possibility to solve this operator equation by average it over all possible products of the metric operator $\hat{g}(x_1), \ldots, \hat{g}(x_n)$ which can be written as an infinite set of equations for all Green's functions:

$$\langle Q|\hat{g}(x_1)\cdot\hat{G}_{\mu\nu}|Q\rangle = k^2\langle Q|\hat{g}(x_1)\cdot\hat{T}_{\mu\nu}|Q\rangle, \tag{2}$$

$$\ldots = \ldots, \tag{3}$$

$$\langle Q|\hat{g}(x_1)\cdot\ldots\cdot\hat{g}(x_n)\cdot\hat{G}_{\mu\nu}|Q\rangle = k^2\langle Q|\hat{g}(x_1)\cdot\ldots\cdot\hat{g}(x_n)\cdot\hat{T}_{\mu\nu}|Q\rangle, \tag{4}$$

where $|Q\rangle$ is a quantum state [10,11]. This set of equations cannot be solved analytically. Some proximate methods to solve them were discussed in Refs [10,13], for example, one can decompose the metric operator into a sum of an average metric $g_{\mu\nu}$ and a fluctuating part $\widehat{\delta g}_{\mu\nu}$ [10].

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \widehat{\delta g}_{\mu\nu}. \tag{5}$$

Assuming $\langle\delta\hat{g}_{\mu\nu}\rangle \neq 0$ and ignoring high order fluctuations, we expand the Einstein–Hilbert Lagrangian $L_{\hat{g}} = \frac{1}{2k^2}\sqrt{-\hat{g}}\hat{R}$ in the

* Correspondence to: College of Physical Science and Technology, Hebei University, Baoding 071002, China.
E-mail address: yangrongjia@tsinghua.org.cn.

http://dx.doi.org/10.1016/j.dark.2016.04.007

following manner

$$L_{\hat{g}} = L_{\hat{g}}(g + \delta\hat{g}) \approx L_g(g) + \frac{\delta L_g}{\delta g^{\mu\nu}}\widehat{\delta g}^{\mu\nu}. \tag{6}$$

The expectation value of $\frac{\delta L_g}{\delta g^{\mu\nu}}\widehat{\delta g}^{\mu\nu}$ can be represented as $\left\langle \frac{\delta L_g}{\delta g^{\mu\nu}}\widehat{\delta g}^{\mu\nu}\right\rangle = \frac{\delta L_g}{\delta g^{\mu\nu}}\langle\widehat{\delta g}^{\mu\nu}\rangle = \sqrt{-g}G_{\mu\nu}\langle\widehat{\delta g}^{\mu\nu}\rangle$ [10]. Then the expectation value of Lagrangian (6) has the form

$$\langle L_{\hat{g}}\rangle \approx \frac{1}{2k^2}\sqrt{-g}\left[R + G_{\mu\nu}\langle\delta\hat{g}^{\mu\nu}\rangle\right]. \tag{7}$$

Similarly, we can expand the quantum Lagrange density $L_{\rm m}^{\hat{g}}$ as follows

$$L_{\rm m}^{\hat{g}}(g + \delta\hat{g}) \approx \sqrt{-g}L_{\rm m}(g) + \frac{\delta\sqrt{-g}L_{\rm m}}{\delta g^{\mu\nu}}\widehat{\delta g}^{\mu\nu}. \tag{8}$$

With the aforementioned assumptions, the expectation value of the quantum Lagrange density (8) takes the form [10]

$$\langle L_{\rm m}^{\hat{g}}(g + \delta\hat{g})\rangle \approx \sqrt{-g}\left[L_{\rm m} + \frac{1}{2}T_{\mu\nu}\langle\delta\hat{g}^{\mu\nu}\rangle\right]. \tag{9}$$

Therefore the modified Lagrangian density can be written as (for details, see Refs. [10,11])

$$L = \frac{1}{2k^2}\sqrt{-g}\left[R + G_{\mu\nu}\langle\delta\hat{g}^{\mu\nu}\rangle\right] + \sqrt{-g}\left[L_{\rm m} + \frac{1}{2}T_{\mu\nu}\langle\delta\hat{g}^{\mu\nu}\rangle\right]. \tag{10}$$

In this paper, we will consider the simplest case: $\langle\delta g^{\mu\nu}\rangle = \alpha g^{\mu\nu}$ and derive the equation of motion. Then we will investigate the effects of quantum fluctuations of metric on the evolution of the universe. Obviously, if $|\alpha| \ll 1$, the modified Lagrangian density (10) approximates to the Hilbert–Einstein Lagrangian density, namely the model of modified gravity proposed here approximates to general relativity. In order to adequately analyze the effects of quantum fluctuations on the universe, or in other words, to take seriously the way to modify gravity from the nonperturbative quantization of a metric [10], we will consider all possible values the parameter α can take.

The paper is outlined as follows. In next section, we will present the modified gravitational field equations and the modified conservational equations. In Section 3, effects of quantum fluctuations on the universe are discussed. Finally, we will briefly summarize and discuss our results in Section 4.

2. Modified gravitational field equations

For Lagrangian density (10), since $\langle\delta\hat{g}_{\mu\nu}\rangle$ has the same symmetry as the metric $g_{\mu\nu}$, the simplest case is $\langle\delta\hat{g}^{\mu\nu}\rangle = \alpha g^{\mu\nu}$, which was suggested in [10]. Therefore Lagrangian density (10) takes the form

$$L = L_{\rm mg} + L_{\rm mm} = \frac{1}{2k^2}\sqrt{-g}(1-\alpha)R + \sqrt{-g}\left[L_{\rm m} - \frac{1}{2}\alpha T\right], \tag{11}$$

where $T = g_{\mu\nu}T^{\mu\nu}$ with $T_{\mu\nu} = -2\delta(\sqrt{-g}L_{\rm m})/(\sqrt{-g}\delta g^{\mu\nu})$. Lagrangian density (11) may can be rewritten as a special type of $f(R,T)$ gravity phenomenologically proposed in [14], but here we obtain it basing on theoretical considerations and for the first time discuss it in detail. In general, comparing with the classical part of the metric, quantum fluctuations are small ($|\alpha| < 1$), such as in radiation, matter, or dark energy dominated era; however, it is possible that quantum fluctuations are large ($|\alpha| > 0$) when the system we consider closes to the Planck scale, such as in the very early epoch. Varying Lagrangian density (10) with respect to $g^{\mu\nu}$ and assuming $\delta g_{\mu\nu} = 0$ on the boundary, we obtain

$$\delta L_{\rm mg} = \frac{1}{2k^2}(1-\alpha)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\delta g^{\mu\nu}, \tag{12}$$

$$\delta(\sqrt{-g}L_{\rm m}) = -\frac{1}{2}T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}, \tag{13}$$

and

$$\begin{aligned}\delta(\sqrt{-g}T) &= T\delta\sqrt{-g} + \sqrt{-g}\delta T \\ &= -\frac{1}{2}g_{\mu\nu}T\sqrt{-g}\delta g^{\mu\nu} + \sqrt{-g}(T_{\mu\nu} + \theta_{\mu\nu})\delta g^{\mu\nu},\end{aligned} \tag{14}$$

where $\delta T/\delta g^{\mu\nu} = T_{\mu\nu} + \theta_{\mu\nu}$ with $\theta_{\mu\nu} = g^{\alpha\beta}\delta T_{\alpha\beta}/\delta g^{\mu\nu}$. Assuming that the Lagrangian density $L_{\rm m}$ of matter depends only on the metric tensor component $g_{\mu\nu}$, not on its derivative, one can easily get $T_{\mu\nu} = g_{\mu\nu}L_{\rm m} - 2\partial L_{\rm m}/\partial g^{\mu\nu}$ and

$$\begin{aligned}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} &= -g_{\alpha\lambda}g_{\beta\sigma}\delta^{\lambda\sigma}_{\mu\nu}L_{\rm m} + \frac{1}{2}g_{\mu\nu}g_{\alpha\beta}L_{\rm m} - \frac{1}{2}T_{\mu\nu}g_{\alpha\beta} \\ &\quad - 2\frac{\partial^2 L_{\rm m}}{\partial g^{\alpha\beta}\partial g^{\mu\nu}},\end{aligned} \tag{15}$$

where $\delta^{\lambda\sigma}_{\mu\nu} = \delta g^{\lambda\sigma}/\delta g^{\mu\nu}$. Then we have

$$\theta_{\mu\nu} = g_{\mu\nu}L_{\rm m} - 2T_{\mu\nu} - 2g^{\alpha\beta}\frac{\partial^2 L_{\rm m}}{\partial g^{\alpha\beta}\partial g^{\mu\nu}}. \tag{16}$$

For different matter, $\theta_{\mu\nu}$ takes different form. From Eqs. (12)–(14), we obtain the gravitational field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{2k^2}{1-\alpha}\left[\frac{1}{2}(1+\alpha)T_{\mu\nu} - \frac{1}{4}\alpha g_{\mu\nu}T + \frac{1}{2}\alpha\theta_{\mu\nu}\right]. \tag{17}$$

We can seen that the gravitational constant G and the energy–momentum tensor are modified due to the quantum fluctuation of the metric. Because $R = -k^2[T + \alpha\theta/(1-\alpha)]$, the gravitational field equations (17) can be rewritten as

$$R_{\mu\nu} = \frac{2k^2}{1-\alpha}\left[\frac{1}{2}(1+\alpha)T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T + \frac{1}{2}\alpha\theta_{\mu\nu} - \frac{1}{4}\alpha g_{\mu\nu}\theta\right]. \tag{18}$$

Taking into account the covariant divergence of Einstein tensor $\nabla_\nu G^{\mu\nu} = 0$, we get for the divergence of the stress–energy tensor $T_{\mu\nu}$ the equation

$$\nabla^\nu T_{\mu\nu} = \frac{1}{1+\alpha}\left[\frac{1}{2}\alpha\nabla_\mu T - \alpha\nabla^\nu\theta_{\mu\nu}\right]. \tag{19}$$

In other words, if we take into account of quantum fluctuations, the stress–energy tensor are not conserved quantities any more. Discussions on the nonconservation of stress–energy tensor can be found in [15–18]. Obviously, for small α, the effects of quantum fluctuations are weak, Lagrangian density (10) approximates to the Hilbert–Einstein Lagrangian density. In order to adequately investigate the effects of quantum fluctuations of metric on the universe, see, for example, for a $\alpha \sim 1$, the effects of quantum fluctuations can even approximatively counteract the gravity, it is worth considering all possible values the parameter α can take. We will discuss this topic in detail in the following sections.