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An interacting dark energy model with nonminimal derivative coupling

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ABSTRACT

We study cosmological dynamics of an extended gravitational theory that gravity is coupled nonminimally with derivatives of a dark energy component and there is also a phenomenological interaction between the dark energy and dark matter. Depending on the direction of energy flow between the dark sectors, the phenomenological interaction gets two different signs. We show that this feature affects the existence of attractor solution, the rate of growth of perturbations and stability of the solutions. By considering an exponential potential as a self-interaction potential of the scalar field, we obtain accelerated scaling solutions that are attractors and have the potential to alleviate the coincidence problem. While in the absence of the nonminimal derivative coupling there is no attractor solution for phantom field when energy transfers from dark matter to dark energy, we show an attractor solution exists if one considers an explicit nonminimal derivative coupling for phantom field in this case of energy transfer. We treat the cosmological perturbations in this setup with details to show that with phenomenological interaction, perturbations can grow faster than the minimal case.

1. Introduction

Observational data such as the type Ia Supernovae redshift-distance surveys [1–7], the Baryon Acoustic Oscillations of the matter density power spectrum [8,6,7] and the angular location of the first peak in the CMB power spectrum [9,10,6,7] from various origins show that the universe currently is experiencing a positive accelerating phase of expansion. To describe this expansion, one can modify the gravitational sector [11–13] or modify the content of the universe by introducing a dark energy component with negative pressure that violates the strong energy condition. The cosmological constant with EoS $\omega = -1$ is the simplest model of dark energy that coincides extraordinarily with observational data, but it suffers from lake of dynamics and fine-tuning problems [14–17].

The dark energy scenario can be described by various scalar fields with variety of dynamical equation of state, among them we can mention quintessence field (a canonical scalar field) [18–21], phantom field (a scalar field with negative kinetic term) [22–28], a combination of both these fields in a unified model called the quintom field model [29–31], tachyon fields that emerge from string theory [32–35], k-essence fields (with a generalized kinetic energy term) [36,37] and Chaplygin gas component [38,39]. Furthermore, there are more complex models that describe the dark energy, in which the fields are non-minimally coupled to the background curvature. Application of these extended scenarios, dubbed "scalar–tensor" theories [40–47], have interesting cosmological outcomes in both inflation and the dark energy eras. As it was shown in [48–51], non-renormalizable operators coming out from the non-minimal coupling violate the unitarity bound of the theory during inflation era. To avoid this unitarity violation and also to find a framework that the Higgs boson would behave like a primordial Inflation, one can consider non-minimal coupling between the derivatives of the scalar fields and curvature [52–57]. This scenario can be regarded as a subset of the most general scalar–tensor theories. In Refs. [58,59] coupling between the scalar field and the kinetic term has been considered as a source of dark energy, and the role of this coupling in the late-time cosmic speed up has been investigated. These theories emerge as low energy limit of some higher dimensional theories, like superstring theory [60] and also appear as part of the Weyl anomaly in $N = 4$ conformal supergravity [61,24]. Furthermore, from a perturbative viewpoint, a new window has been opened on the issue of quantum gravity proposal in this framework [62]. The role of this non-minimal derivative coupling during inflation has been considered in Refs. [63–65].

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From another perspective, possible interaction between the dark energy and dark matter opens new window on the issue of the cosmological coincidence problem. Although there is no direct evidence for interaction between the dark sectors at least currently, in the absence of a fundamental theory that excludes interaction between the dark sectors, we can consider non-minimal interaction between dark energy and dark matter to alleviate coincidence problem [66–72]. Moreover this interaction potentially improves interpretation of observational data [73–75]. Therefore, it is important, at least theoretically, to see possible outcomes of such an interaction and its impact on late time cosmological dynamics. For this reason we include also an interaction, much on the basis of some phenomenological considerations, between the dark sectors with the hope to shed some light on the issue of cosmological coincidence problem. By considering such an interaction between the dark sectors, whether the energy flows from dark matter to the dark energy or the reverse occurs, now is an important issue in late time cosmic dynamics. The direction of energy flow due to interaction between the dark sectors affects considerably the issues such as the existence of attractor solutions, growth rate of perturbations and the stability of cosmological solutions. With these points in mind, we consider two different candidates for dark energy: a quintessence and a phantom field, and in each case we analyze the cosmological dynamics in phase space, the statefinder diagnostic, stability in $w-w'$ phase plane and the full analysis of the perturbations in this setup with some exact solutions. The behavior of these solutions for matter perturbations on sub-Hubble scales are treated carefully for matter and scaling solutions eras.

2. The model

We consider an extension of scalar–tensor theories of gravity that derivatives of a scalar field, as a dark energy candidate, are coupled to curvature and there is also a phenomenological interaction between the dark energy and dark matter components. Our final goal with these types of extension is to see the status of coincidence problem and also growth rate of perturbations in this setup. Following the pioneer work of Amendola [52], the Lagrangian of possible interaction between gravity and derivatives of the dark energy component can be sorted as follows

$$L_1 = k_1 R\varphi_{,\mu}\varphi^{,\mu}, \qquad L_2 = k_2 R_{\mu\nu}\varphi^{,\mu}\varphi^{,\nu}, \qquad L_3 = k_3 R\Box\varphi,$$
$$L_4 = k_4 R_{\mu\nu}\varphi\varphi^{;\mu\nu}, \qquad L_5 = k_5 R_{;\mu}\varphi\varphi^{,\mu}, \qquad L_6 = k_6\Box R\varphi^2.$$

(For more details see also [54].) Here we just consider L_1 and L_2 since, as discussed in [52,53,55,56,63], using total divergences and without loss of generality one can keep only the first two terms. The coefficients k_1 and k_2 are coupling parameters with dimension of length-squared. As a specific case, and more importantly in order the resulting theory to be free of ghosts (see for instance [56]), we set $k_2 = -2k_1 = \eta$, which gives the Einstein tensor $G_{\mu\nu}$. Therefore, the ghost-free action of our setup takes the following form

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\epsilon g_{\mu\nu} - \eta G_{\mu\nu})\partial^\mu\varphi\partial^\nu\varphi - V(\varphi) - F(\varphi)\mathcal{L}_m\right] \tag{1}$$

where $F(\varphi) = \beta e^{\alpha\varphi}$ is the interacting term between the dark sectors with constants α and $\beta > 0$, and R is the curvature scalar, φ is the homogeneous scalar field (as a dark energy component), $V(\varphi)$ is the scalar field potential and $\mathcal{L}_m$ is the Lagrangian density of matter (all sorts of matter except baryons and radiation which are subdominant and supposed to be minimally coupled to gravity). We consider the system of units in which $8\pi G = c = \hbar = 1$. In addition, we use a symbol ϵ in order to show quintessence and phantom field in a unified manner so that ϵ takes the value $+1$ for the quintessence field and -1 for the phantom field. By taking variation of the action (1) with respect to the metric, we get the field equations [55,56] as follows

$$G_{\mu\nu} = \epsilon T^{(\varphi)}_{\mu\nu} + \eta T^{(\eta)}_{\mu\nu} + T^{(m)}_{\mu\nu} - g_{\mu\nu}V(\varphi) \tag{2}$$

with

$$T^{(\varphi)}_{\mu\nu} = \nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{2}g_{\mu\nu}(\nabla\varphi)^2, \tag{3}$$

$$\begin{aligned} -T^{(\eta\varphi)}_{\mu\nu} = {} & -\frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi R + 2\nabla_\alpha\varphi\nabla_{(\mu}\varphi R^\alpha_{\nu)} + \nabla^\alpha\varphi\nabla^\beta\varphi R_{\mu\alpha\nu\beta} \\ & + \nabla_\mu\nabla^\alpha\varphi\nabla_\nu\nabla_\alpha\varphi - \nabla_\mu\nabla_\nu\varphi\Box\varphi - \frac{1}{2}(\nabla\varphi)^2 G_{\mu\nu} \\ & + g_{\mu\nu}\left[-\frac{1}{2}\nabla^\alpha\nabla^\beta\nabla_\alpha\nabla_\beta\varphi + \frac{1}{2}(\Box\varphi)^2 - \nabla_\alpha\varphi\nabla_\beta\varphi R^{\alpha\beta}\right] \end{aligned} \tag{4}$$

where $\nabla_{(\mu}\varphi R^\alpha_{\nu)} = \frac{1}{2}(\nabla_\mu\varphi R^\alpha_\nu + \nabla_\nu\varphi R^\alpha_\mu)$ and $T^{(\varphi)}_{\mu\nu}$, $T^{(\eta)}_{\mu\nu}$ correspond to the variation of the terms that depend on the scalar field in the Jordan frame and $T^{(m)}_{\mu\nu}$ is the ordinary energy–momentum tensor of matter component. Considering a spatially-flat Friedmann–Robertson–Walker metric as

$$\begin{aligned} & ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2), \\ & d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2 \end{aligned} \tag{5}$$

where t is the cosmic time, (r, θ, φ) are the comoving spatial (radial and angular) coordinates, $a(t)$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter, the field equations (3) and (4) for (00) and (11) components (energy density and pressure, respectively) take the following form

$$\rho_\varphi = \epsilon\frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{9}{2}\eta H^2\dot{\varphi}^2, \tag{6}$$

$$p_\varphi = \epsilon\frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \eta\left(\dot{H}\dot{\varphi}^2 + \frac{3}{2}H^2\dot{\varphi}^2 + 2H\dot{\varphi}\ddot{\varphi}\right). \tag{7}$$

Friedmann equations can be written as

$$3H^2 = F(\varphi)\rho_m + \frac{1}{2}\dot{\varphi}^2(\epsilon + 9\eta H^2) + V(\varphi), \tag{8}$$

$$\begin{aligned} \dot{H}\left(1 - \frac{1}{2}\eta\dot{\varphi}^2\right) = {} & -\epsilon\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\gamma F(\varphi)\rho_m \\ & - \eta\left(\frac{3}{2}H^2\dot{\varphi}^2 - H\dot{\varphi}\ddot{\varphi}\right) \end{aligned} \tag{9}$$

where $\gamma \equiv 1 + w_m$ is the barotropic index which depends on the type of matter. Variation of the action (1) with respect to the scalar field gives the equation of motion of this field as

$$\begin{aligned} & \epsilon(\ddot{\varphi} + 3H\dot{\varphi}) + 3\eta(H^2\ddot{\varphi} + 2\dot{\varphi}H\dot{H} + 3H^3\dot{\varphi}) + V'(\varphi) \\ & = F'(\varphi)\rho_m, \end{aligned} \tag{10}$$

where a prime represents derivative with respect to φ. The continuity equations for scalar field and dark matter are respectively as follows

$$\dot{\rho}_\varphi + 3H(1 + \omega_\varphi)\rho_\varphi = Q, \tag{11}$$

$$(F(\varphi)\rho_m)^{\cdot} + 3H\gamma F(\varphi)\rho_m = -Q \tag{12}$$

where $Q = F'(\varphi)\dot{\varphi}\rho_m$ is a specific interaction term obtained in this model. The sign of Q shows the direction of energy transfer

$$\begin{cases} Q > 0, & \text{Energy transfers from dark matter to dark energy.} \\ Q < 0, & \text{Energy transfers from dark energy to dark matter.} \end{cases}$$