Physics of the Dark Universe 13 (2016) 111–120

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Constraining the Schwarzschild–de Sitter solution in models of modified gravity

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A R T I C L E I N F O

Article history:
Received 3 March 2016
Received in revised form
12 May 2016
Accepted 16 May 2016

Keywords:
Experimental studies of gravity
Modified gravity
Dark energy
Lunar, planetary, and deep-space probes

A B S T R A C T

The Schwarzschild–de Sitter (SdS) solution exists in the large majority of modified gravity theories, as expected, and in particular the effective cosmological constant is determined by the specific parameters of the given theory. We explore the possibility to use future extended radio-tracking data from the currently ongoing New Horizons mission in the outskirts peripheries of the Solar System, at about 40 au, in order to constrain this effective cosmological constant, and thus to impose constrain on each scenario's parameters. We investigate some of the recently most studied modified gravities, namely $f(R)$ and $f(T)$ theories, dRGT massive gravity, and Hořava–Lifshitz gravity, and we show that New Horizons mission may bring an improvement of one-two orders of magnitude with respect to the present bounds from planetary orbital dynamics.

1. Introduction

General Relativity (GR) has undergone brilliant successes since its inception 100 years ago (see, e.g., the review [1] and references therein). Einstein's theory is the standard paradigm for describing the gravitational interaction, verified by many experimental evidences [2], even though, with the possible exception of binary-pulsar systems, at least to a certain extent, these tests are probes of the weak-field gravity, or differently speaking they probe gravity up to intermediate scales ($\simeq 1 - 10^1$ au). Nevertheless, one of the current challenges in theoretical physics and cosmology is the description of gravitation at large scales. In particular, evidences from astrophysics and cosmology [3–12] suggest that the Universe content is 76% dark energy, 20% dark matter, 4% ordinary baryonic matter. This implies that in order to reconcile the observations with GR we are led to assume that the Universe is dominated by dark entities, with peculiar characteristics. The dark energy is an exotic cosmic fluid, which has not yet been detected directly, and which does not cluster as ordinary matter; indeed, its behavior closely resembles that of the cosmological constant Λ, which, in turn, brings about other problems, concerning its nature and origin [13,14]. On the other hand, the dark matter is an unknown type of matter, which has the clustering properties of ordinary matter; since 1933 it is has been related to the problem of missing matter in astrophysical scenarios [15]. Moreover, some kind of cold and pressureless dark matter (whose distribution is that of a spherical halo around the galaxies) is also required to explain the rotation curves of spiral galaxies [16]. Hence, the best answer we have today for these cosmic puzzles is the so called concordance model or ΛCDM, which provides the simplest description of the available data concerning the large-scale structure of the Universe. For a recent review, see e.g. [17]. This picture is completed with the inflationary scenario which solves the horizon, flatness and monopole problems [18].

Besides these difficulties in explaining observations, there are theoretical motivations suggesting that a theory of gravity more fundamental than GR should be formulated: Einstein's theory is not renormalizable, and thus it cannot be quantized as is. In

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http://dx.doi.org/10.1016/j.dark.2016.05.001
2212-6864/

a recent paper by Berti et al. [19], a thorough review of the motivations to consider extensions of GR can be found together with a discussion of some modified theories of gravity (see also the recent reviews [20–24] and references therein).

A possible strategy towards a new theory of gravity is, in some sense, a natural generalization of Einstein's approach, according to which gravity is geometry. Accordingly, a new theory is obtained extending GR on a purely geometric basis: in other words, the required ingredients to match the observations or to solve the theoretical conundrums derive from a geometric structure richer than that of GR.

As a prototype of this strategy, which has gained an increasing attention during the last decade, we mention the $f(R)$ theories, where the gravitational Lagrangian depends on a function of the scalar curvature R; extensive reviews can be found in [25–29]. These theories are also referred to as "extended theories of gravity", since they naturally generalize GR: in fact, when $f(R) = R$ the action reduces to the usual Einstein–Hilbert action, and Einstein's theory is obtained.

Motivations for studying these theories can be different but, as clearly synthesized by Sotiriou and Faraoni [27], they can be considered as toy-theories that are relatively simple to handle and that allow to study the effects of the deviations from Einstein's theory with sufficient generality. For instance $f(R)$ theories provide cosmologically viable models, where both the inflation phase and the late-time accelerated expansion are reproduced; furthermore, they have been used to explain the rotation curves of galaxies without need for dark matter (see [25,27,28] and references therein). These theories can be studied in the metric formalism, where the action is varied with respect to metric tensor, and in the Palatini formalism, where the action is varied with respect to the metric and the affine connection, which are supposed to be independent from one another (there is also the metric-affine formalism, in which the matter part of the action depends on the affine connection, and is then varied with respect to it). In general, the two approaches are not equivalent: the solutions of the Palatini field equations are a subset on solutions of the metric field equations [30].

A different approach to the extension of GR derives from a generalization of Teleparallel Gravity (TEGR) [31,32]: this theory is based on a Riemann–Cartan space–time, endowed with the non symmetric Weitzenböck connection which, unlike the Levi-Civita connection of GR, gives rise to torsion but it is curvature-free. In TEGR torsion determines the geometry, while the tetrad field is the dynamical one; the field equations are obtained from a Lagrangian containing the torsion scalar T, arising from contractions of the torsion tensor. Notwithstanding GR and TEGR have a different geometric structure, they have the same dynamics: in other words, every solution of GR is also solution of TEGR and vice versa. Hence, one could start from TEGR and extend its Lagrangian from T to an arbitrary function $f(T)$, resulting to the so-called $f(T)$ gravity [33,34] (for a review see [35]). Since $f(T)$ gravity is different from TEGR, $f(T)$ theories have been considered as potential candidates to describe the cosmological behavior [36–47]. Additionally, various aspects of $f(T)$ gravity have been considered, such as for instance, exact solutions and stellar models [48–60].

Another possible new theory of gravity can be obtained by a massive deformation of GR. Endowing graviton with a mass is a plausible modified theory of gravity that is both phenomenologically and theoretically intriguing. From the theoretical point of view, a small non-vanishing graviton mass is an open issue. The idea was originally introduced in the work of Fierz and Pauli [61], who constructed a massive theory of gravity in a flat background that is ghost - free at the linearized level. Since then, a great effort has been put in extending the result to the nonlinear level and constructing a consistent theory. A few years ago a covariant massive gravity model has been proposed in [62]. Since the linearization of the mass term breaks the gauge invariance of GR then, in order to construct a consistent theory, non-linear terms should be tuned to remove order by order the negative energy state in the spectrum [63]. The theoretical model under investigation follows from a procedure originally outlined in [64,65] and has been found not to show ghosts at least up to quartic order in the nonlinearities [62,66]. The consequent theory exploits several remarkable features. Indeed the graviton mass typically manifests itself on cosmological scales at late times thus providing a natural explanation of the presently observed accelerating phase [67]. Moreover, the theory allows for exotic solutions in which the contribution of the graviton mass affects the dynamics at early times. It actually allows for models in which the Universe oscillates indefinitely about an initial static state, ameliorating the fine-tuning problem suffered by the emergent Universe scenario in GR [68].

Another approach that could lead towards a formulation of a quantum theory of gravity is the Hořava formulation of a model that is power-counting renormalizable due to an anisotropic scaling of space and time [69]. This is reminiscent of Lifshitz scalars in condensed matter physics [70,71], hence the theory is often referred to as the Hořava–Lifshitz gravity. This theory has attracted a lot of attention, due to its several remarkable features in cosmology. Unfortunately, the original model suffers from instability, ghosts, strong coupling problems and the model has been implemented along different lines [72].

On the other hand, if we consider the excellent agreement of GR with Solar System and binary pulsar observations, it is apparent that any modified theory of gravity should reproduce GR at the Solar System scale, i.e. in a suitable weak-field limit. In other words, these theories must have correct Newtonian and post-Newtonian limits and, up to intermediate scales, the deviations from the GR predictions can be considered as perturbations. This agreement should be obtained, for all the above gravitational modifications. In particular, all these theories have the same spherically symmetric solution that describes the gravitational field around a point-like source: the Schwarzschild–de Sitter space–time (SdS). Interestingly enough, this is a solution of GR field equations with a cosmological constant. However, for these modified gravities the cosmological term is not added by hand, but it naturally originates from the modified Lagrangian.

In this paper, we assume that the SdS solution can be used to model the gravitational field of an isolated source like the Sun, and we examine the impact that the gravitational modifications have on the Solar System dynamics. Additionally, we explore the possibility of constraining Λ in the distant peripheries of the Solar System by means of the currently ongoing spacecraft-based mission New Horizons. For a recently proposed long-range mission aimed to test long-distance modifications of gravity in the Solar System, see [73].

This work is organized as follows: In Section 2 we describe the main features of the SdS space–time, focusing on $f(R)$ and $f(T)$ theories, massive gravity and Hořava–Lifshitz gravity. Section 3 is devoted to a preliminary exposition of the experimental constraints which might be posed by using accurate tracking of distant man-made objects traveling to the remote outskirts of the Solar System; the case of the New Horizons probe is considered. Finally, Section 4 summarizes our results.

2. Schwarzschild–de Sitter space–time as a vacuum solution of modified gravities

The SdS metric (see e.g [74])

$$ds^2 = \left(1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 - \frac{1}{\left(1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2\right)} dr^2 - r^2 d\Omega^2 \quad (1)$$