Physics of the Dark Universe 11 (2016) 11–15

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Turnaround radius in modified gravity

Valerio Faraoni

Physics Department and STAR Research Cluster, Bishop's University, Sherbrooke, Québec, Canada J1M 1Z7

ARTICLE INFO

Article history:
Received 10 August 2015
Received in revised form
26 September 2015
Accepted 3 November 2015

Keywords:
Modified gravity
Dark energy theory
Dark energy experiments
Galaxy clusters

ABSTRACT

In an accelerating universe in General Relativity there is a maximum radius above which a shell of test particles cannot collapse, but is dispersed by the cosmic expansion. This radius could be used in conjunction with observations of large structures to constrain the equation of state of the universe. We extend the concept of turnaround radius to modified theories of gravity for which the gravitational slip is non-vanishing.

1. Introduction

Since 1998 cosmologists and theoretical physicists have been trying to explain the present acceleration of the universe discovered with type Ia supernovae. Apart from the problematic cosmological constant, explanations based on a dark energy introduced *ad hoc* (see [1] for a review) are not satisfactory and many researchers have turned to contemplating the possibility of modifying gravity at large scales ([2], see [3–7] for reviews). Although modifying gravity is a viable possibility, too many dark energy and modified gravity models fit the observational data and it is important to use any test of gravity which may become available, at all scales, to obtain hints on the correct explanation of the cosmic acceleration and, possibly, on the correct theory of gravity.

One possibility which has been pointed out recently is testing theoretical predictions of the turnaround radius with astronomical observations [8–25]. In an accelerating Friedmann–Lemaître–Robertson–Walker (FLRW) universe, there is a maximum (areal) radius beyond which a spherical shell of dust cannot collapse but expands forever, driven by the cosmic accelerated expansion. We formulate our final results in terms of the areal radius (R) because solutions of modified gravity theories are reported in the literature using various radial coordinates. However, effects peculiar to a particular coordinate system would not be meaningful in relativistic gravity,[1] while effects characterized in a geometric, coordinate-independent, way are physically meaningful. The areal radius separating two points in space is a physical distance identified in a completely geometric way by the area of 2-spheres of symmetry in a spherically symmetric spacetime. In a spatially FLRW universe with line element $ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2_{(2)}\right)$, the physical (areal) radius is $R(t, r) = a(t)r$ and it expands with the scale factor $a(t)$, while the comoving radius r is simply a label attached to elements of the cosmic fluid.

The first comparisons of the prediction for the turnaround radius in the ΛCDM model of General Relativity with objects in the sky have been carried out [23–25,22] and, although the precision is still poor, the method holds promise. In General Relativity, the concept of turnaround radius can be made more rigorous by using the Hawking–Hayward quasi-local mass [26]. Given the motivation for modified gravity in cosmology, here we propose to extend the scope of studies of the turnaround radius to alternative theories of gravity. We do not commit to any specific modified gravity theory at this stage, but adopt a post-Friedmannian approach [27] in which a post-FLRW metric fits many theories, to lowest order in metric perturbations from an exact FLRW background. Contrary to General Relativity, in which two scalar potentials in the perturbed FLRW metric coincide (apart from the sign), in modified gravity there are two distinct potentials which are not trivially related. We derive the turnaround radius in this scheme and find that the physical (areal) turnaround radius depends on both potentials. We point out that, in order for studies of the turnaround radius to be meaningful, it is not sufficient to pick a theory of modified gravity but efforts must be made to establish which solutions of this theory are generic in some appropriate sense.

The metric signature employed in this paper is $-+++$ and we use units in which Newton's constant G and the speed of light c assume the value unity.

E-mail address: vfaraoni@ubishops.ca.

[1] Unless that coordinate system is associated with a preferred family of observers.

http://dx.doi.org/10.1016/j.dark.2015.11.001

2. Turnaround radius in the parametrized post-Friedmannian approach

The parametrized post-Friedmannian approach [27] describes perturbations of a FLRW universe in theories of gravity alternative to General Relativity. The line element (1) is a rather general parametrization of the metric describing perturbed FLRW universes in modified gravity [28–30] (it holds, for example, in $f(R)$ gravity [31,32]). The spacetime metric in the conformal Newtonian gauge is [33–38]

$$ds^2 = a^2(\eta)\left[-(1+2\psi)\,d\eta^2 + (1-2\phi)\left(dr^2 + r^2 d\Omega^2_{(2)}\right)\right], \quad (1)$$

where $d\Omega^2_{(2)} = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the metric on the unit 2-sphere, η is the conformal time (related to the comoving time t by $dt = a d\eta$), $a(\eta)$ is the scale factor of the spatially flat FLRW background, and ϕ and ψ are two post-Friedmannian scalar potentials. While in General Relativity it is $\psi = -\phi$, in many modified theories of gravity these two potentials do not coincide in absolute value and the gravitational slip $\xi \equiv (\phi - \psi)/\phi$ is used in experiments aiming at detecting deviations from the standard ΛCDM scenario [39,40,33,35–38]. The ansatz (1) does not include vector and tensor metric perturbations, which is justified at lower order for non-relativistic velocities, and is common practice in the literature on cosmological perturbations (e.g., [41,42]).[2]

Following the literature on the turnaround radius in cosmology [23–25,22], we assume that the FLRW perturbation is spherically symmetric, *i.e.*, $\phi = \phi(r)$, $\psi = \psi(r)$. The numerical importance of deviations from spherical symmetry was discussed in [43,23,24]. The simplest definition of turnaround radius consists of considering spherical shells of test particles respecting the spacetime symmetry (that is, expanding or contracting but not shearing nor rotating) with areal radius R, and imposing that they have zero radial acceleration, $\ddot{R} = 0$, where an overdot denotes differentiation with respect to the comoving time t of the FLRW background. We will adopt here this common criterion which, in General Relativity, can be justified rigorously [44] by making use of the Hawking–Hayward quasi-local mass construct [45,46]. In general, this quasi-local energy construct is not defined in modified gravity and here we will stick to the simple definition $\ddot{R} = 0$ for shells of test particles (dust) in radial motion.

Massive test particles follow timelike geodesics with 4-tangents u^a satisfying $u_c u^c = -1$ and the geodesic equation

$$\frac{du^a}{d\tau} + \Gamma^a_{bc} u^b u^c = 0, \quad (2)$$

which we choose to be affinely parametrized by the proper time τ, and where

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad}\left(g_{db,c} + g_{dc,b} - g_{bc,d}\right) \quad (3)$$

are the coefficients of the metric connection. We will perform first order calculations in the metric perturbations ϕ and ψ, but the density contrast related to the spatial Laplacian of these metric perturbations is allowed to be large [44,41,42]. The normalization

$$u_c u^c = g_{00}(u^0)^2 + g_{11}(u^1)^2 = -1 \quad (4)$$

for massive test particles with purely radial motion ($u^2 = u^3 = 0$) yields

$$(u^0)^2 = \frac{1}{a^2}(1-2\psi) + (1-2\phi-2\psi)(u^1)^2 \quad (5)$$

to first order. Eq. (2) then gives

$$\frac{du^\mu}{d\tau} + \Gamma^\mu_{00}(u^0)^2 + 2\Gamma^\mu_{01} u^0 u^1 + \Gamma^\mu_{11}(u^1)^2 = 0 \quad (6)$$

for $\mu = 0, 1$. Eq. (3) provides the only non-vanishing Christoffel symbols to first order

$$\Gamma^0_{00} = \frac{a_\eta}{a}, \qquad \Gamma^0_{01} = \Gamma^0_{10} = \psi', \qquad \Gamma^0_{11} = \frac{a_\eta}{a}(1-2\phi-2\psi), \quad (7)$$

$$\Gamma^1_{00} = \psi', \qquad \Gamma^1_{01} = \Gamma^1_{10} = \frac{a_\eta}{a}, \qquad \Gamma^1_{11} = -\phi', \quad (8)$$

where a prime denotes differentiation with respect to the radial coordinate r and $a_\eta \equiv da/d\eta$. Therefore, it is

$$\frac{du^0}{d\tau} + \frac{a_\eta}{a}(u^0)^2 + 2\psi' u^0 u^1 + \frac{a_\eta}{a}(1-2\phi-2\psi)(u^1)^2 = 0, \quad (9)$$

$$\frac{du^1}{d\tau} + \psi'(u^0)^2 + \frac{2a_\eta}{a} u^0 u^1 - \phi'(u^1)^2 = 0. \quad (10)$$

The areal radius of the spherical spacetime (a geometric and coordinate-independent quantity) is

$$R(t,r) = ar\sqrt{1-2\phi} \simeq ar(1-\phi) \quad (11)$$

to first order. Since

$$\frac{dR}{dt} = (\dot{a}r + a\dot{r})(1-\phi), \quad (12)$$

$$\frac{d^2R}{dt^2} = (\ddot{a}r + 2\dot{a}\dot{r} + a\ddot{r})(1-\phi), \quad (13)$$

and

$$\dot{r} \equiv \frac{dr}{dt} = \frac{dr}{d\eta}\frac{d\eta}{dt} = \frac{1}{a}\frac{dr}{d\tau}\frac{d\tau}{d\eta} = \frac{u^1}{au^0}, \quad (14)$$

$$\ddot{r} = \frac{d}{dt}\left(\frac{u^1}{au^0}\right) = -\frac{\dot{a}}{a^2}\frac{u^1}{u^0} + \frac{1}{a^2u^0}\frac{d}{d\tau}\left(\frac{u^1}{u^0}\right), \quad (15)$$

we have

$$\frac{d^2R}{dt^2} = \left[\ddot{a}r + \frac{\dot{a}u^1}{au^0} + \frac{1}{au^0}\frac{d}{d\tau}\left(\frac{u^1}{u^0}\right)\right](1-\phi). \quad (16)$$

The criterion $d^2R/dt^2 = 0$ locating the turnaround radius [23–25] becomes

$$\ddot{a}r + H\frac{u^1}{u^0} + \frac{1}{au^0}\left[\frac{1}{u^0}\frac{du^1}{d\tau} - \frac{u^1}{(u^0)^2}\frac{du^0}{d\tau}\right] = 0, \quad (17)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter in comoving time. To zero order we have $R = ar$ and $\dot{R} = \dot{a}r + a\dot{r} = HR$ (the Hubble law) for timelike geodesics, so that their 4-tangents have components

$$u^\mu = u^\mu_{(0)} + \delta u^\mu = \left(\frac{1}{a}, 0, 0, 0\right) + \delta u^\mu \quad (18)$$

in coordinates $(\eta, r, \theta, \varphi)$, where the perturbations δu^μ are of first order. To zero order it is $u^0_{(0)} = d\eta/d\tau = d\eta/dt = 1/a$ and $u^1_{(0)} = 0$. Eq. (17) now yields, to first order,

$$\ddot{a}r + 2\dot{a}\delta u^1 + a\frac{d(\delta u^1)}{d\tau} = 0. \quad (19)$$

[2] An effect due to a metric of the form (1) would signal modified gravity and, neglecting vector and tensor degrees of freedom in the metric (which is legitimate at this order of approximation), the potentials ψ and φ are all that remains in the line element. However a better characterization of modified gravity than the metric is, at least in principle, needed and a rigorous derivation of the line element (1) in various classes of modified gravity theories is still missing. Nevertheless, the turnaround radius will probably be unaffected by these theoretical improvements.