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Dark energy as a kinematic effect

H. Jennen *, J.G. Pereira

Instituto de Física Teórica, UNESP - Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 – Bl. II, 01140-070, São Paulo, SP, Brazil

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ABSTRACT

We present a generalization of teleparallel gravity that is consistent with local spacetime kinematics regulated by the de Sitter group $SO(1,4)$. The mathematical structure of teleparallel gravity is shown to be given by a nonlinear Riemann–Cartan geometry without curvature, which inspires us to build the generalization on top of a de Sitter–Cartan geometry with a cosmological function. The cosmological function is given its own dynamics and naturally emerges nonminimally coupled to the gravitational field in a manner akin to teleparallel dark energy models or scalar–tensor theories in general relativity. New in the theory here presented, the cosmological function gives rise to a kinematic contribution in the deviation equation for the world lines of adjacent free-falling particles. While having its own dynamics, dark energy manifests itself in the local kinematics of spacetime.

1. Introduction

Physically equivalent to general relativity in its description of the gravitational interaction, teleparallel gravity is mathematically and conceptually rather different from Einstein's *opus magnum*. Although the precise implementation of general relativity differs from the one of teleparallel gravity, their geometric structures are related by switching between certain subclasses of Riemann–Cartan spacetimes. On the one hand, general relativity being the standard model for classical gravity, it is naturally well known that the fundamental field is the vierbein, which is accompanied by the Levi-Civita spin connection. The resulting spacetime is thus characterized by a Riemann–Cartan geometry without torsion. On the other hand, teleparallel gravity takes a different route in order to generalize the geometry of Minkowski space to incorporate the dynamics of the gravitational field, for it is torsion instead of curvature that is turned on by gravitating sources [1]. Since the curvature remains zero, the spin connection retains its role of representing inertial effects only. Therefore, the description can be extended to account for a breakdown of the weak equivalence principle [2].

The rationale behind Riemann–Cartan geometry underlying both theories is closely related to the assumption that kinematics is locally governed by the Poincaré group $ISO(1,3)$. Be that as it may, there is significant evidence that our universe momentarily undergoes accelerated expansion [3,4], which indicates that the large-scale kinematics of spacetime is approximated better by the de Sitter group $SO(1,4)$ [5]. We shall take this evidence to heart and conjecture that local kinematics is regulated by the de Sitter group. Looked at from a mathematical standpoint, this amounts to have the Riemann–Cartan geometry replaced by a Cartan geometry modeled on de Sitter space [6]. The corresponding spacetime is everywhere approximated by de Sitter spaces, whose combined set of cosmological constants in general varies from event to event, hence resulting in the cosmological function [7].

In the present article we propose an extended theory of gravity as we generalize teleparallel gravity for such a de Sitter–Cartan geometry. Quite similar to the cosmological constant in teleparallel gravity or general relativity, we model the dark energy driving the accelerated expansion by a cosmological function Λ of dimension one over length squared. Fundamentally different, however, the cosmological function alters the kinematics governing physics around any point, such that spacetime is approximated locally by a de Sitter space of cosmological constant Λ. To be exact, a congruence of particles freely falling in an external gravitational field exhibits a relative acceleration, not only due to the nonhomogeneity of the gravitational field, but also because of the local kinematic properties of spacetime that are determined by the cosmological function.

The organization of the article is as follows. In Section 2 the basic tools for de Sitter–Cartan geometry that are used in subsequent sections are reviewed briefly. Afterwards, we show in Section 3 that the mathematical structure underlying teleparallel gravity is that of a nonlinear Riemann–Cartan geometry, which is consistent with the standard interpretation of teleparallel gravity as a gauge theory for the Poincaré translations. This is an important

* Corresponding author.
E-mail addresses: hjennen@ift.unesp.br (H. Jennen), jpereira@ift.unesp.br (J.G. Pereira).

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observation, since we leave the gauge picture for what it is upon generalizing the theory for a cosmological function in de Sitter teleparallel gravity. Section 4 is devoted to the main results of this article, in which we discuss in sequence the fundamentals of de Sitter teleparallel gravity, the phenomenology of the kinematic effects, and the dynamics of the gravitational field and the cosmological function. We conclude in Section 5.

2. de Sitter–Cartan geometry with a cosmological function

The Cartan geometry modeled on $(\mathfrak{so}(1,4), SO(1,3))$ consists of a principal Lorentz bundle over spacetime together with an $\mathfrak{so}(1,4)$-valued one-form, which is called the Cartan connection [6, 8–10]. The Lie algebra $\mathfrak{so}(1,4)$ that generates the de Sitter group is subject to the commutation relations

$$\begin{aligned}
-i[M_{ab}, M_{cd}] &= \eta_{ac}M_{bd} - \eta_{ad}M_{bc} + \eta_{bd}M_{ac} - \eta_{bc}M_{ad},\\
-i[M_{ab}, P_c] &= \eta_{ac}P_b - \eta_{bc}P_a,\\
-i[P_a, P_b] &= -l^{-2}M_{ab},
\end{aligned} \tag{1}$$

where $\eta_{ab} = (+,-,-,-)$ and we parametrize elements of $\mathfrak{so}(1,4)$ by $\frac{i}{2}\lambda^{ab}M_{ab} + i\lambda^a P_a$, so that the M_{ab} span the Lorentz subalgebra, while the $P_a \equiv M_{a4}/l$ are a basis for the de Sitter transvections. The length scale l is related to the cosmological constant of the corresponding de Sitter space $SO(1,4)/SO(1,3)$, namely, [6]

$$\Lambda = \frac{3}{l^2}. \tag{2}$$

Because the Cartan connection is valued pointwise in copies of (1), the set of length scales $l(x)$ may form an arbitrary function of spacetime [7,11]. Concomitantly, the cosmological constants (2) of the local de Sitter spaces constitute a nonconstant cosmological function Λ.

Corresponding to the reductive nature of the relations (1), the Cartan connection is decomposed in a spin connection $\omega^a{}_{b\mu}$ and vierbein $e^a{}_\mu$, from which it follows that the vierbein has dimension of length. Furthermore, the curvature and torsion of the geometry are given by [7]

$$R^a{}_{b\mu\nu} = B^a{}_{b\mu\nu} + \frac{1}{l^2}(e^a{}_\mu e_{b\nu} - e^a{}_\nu e_{b\mu}) \tag{3}$$

and

$$T^a{}_{\mu\nu} = G^a{}_{\mu\nu} - \partial_\mu \ln l\, e^a{}_\nu + \partial_\nu \ln l\, e^a{}_\mu, \tag{4}$$

where the two-forms

$$B^a{}_{b\mu\nu} = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu} \tag{5}$$

and

$$G^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}e^b{}_\nu - \omega^a{}_{b\nu}e^b{}_\mu \tag{6}$$

are the exterior covariant derivatives of the spin connection, respectively, vierbein. When we solve (6) for the spin connection, the Ricci theorem is recovered:

$$\omega^a{}_{b\mu} = \mathring{\omega}^a{}_{b\mu} + K^a{}_{b\mu}, \tag{7}$$

where $\mathring{\omega}^a{}_{b\mu} = \frac{1}{2}e^c{}_\mu(\Omega_{bc}{}^a + \Omega_b{}^a{}_c + \Omega_c{}^a{}_b)$ is the Levi-Civita spin connection, with $\Omega_{abc} = e_a{}^\mu e_b{}^\nu(\partial_\mu e_{c\nu} - \partial_\nu e_{c\mu})$ the coefficients of anholonomy, and where

$$K^a{}_{b\mu} = \frac{1}{2}(G^a{}_{\mu b} + G_\mu{}^a{}_b + G_b{}^a{}_\mu) \tag{8}$$

is the contortion of $\omega^a{}_{b\mu}$.

Subsequently, it is straightforward to define algebraic covariant differentiation by $D_\mu = \partial_\mu + \omega_\mu$, whereas $\nabla_\mu = \partial_\mu + \Gamma_\mu$ stands for the corresponding spacetime covariant derivative, i.e., $\Gamma^\rho{}_{\nu\mu} = e_a{}^\rho D_\mu e^a{}_\nu$. A metric structure is readily constructed as well by defining $g_{\mu\nu} = e^a{}_\mu e_{a\nu}$, a symmetric tensor that is covariantly constant.

To conclude we remark that the pointwise decomposition of the Cartan connection in a spin connection and vierbein is manifestly local Lorentz invariant. The decomposition, albeit arbitrary, is not unique, for the possibilities are in a bijective correspondence with sections of the fiber bundle $Q[dS]$ of de Sitter spaces, associated with a principal $SO(1,4)$ bundle Q over spacetime [12]. The hidden symmetries, which are generic local de Sitter transformations that do not belong to the Lorentz subgroup, are manifestly restored when the geometric objects are constructed such that they have an explicit dependence on the section chosen. This can be achieved through a nonlinear realization of the Cartan connection and it has been shown in [7] that such a nonlinear de Sitter–Cartan geometry with a cosmological function has a structure identical to the one outlined above. Therefore, we may assume the geometry of this section to be $SO(1,4)$ invariant.

3. Cartan geometric structure of teleparallel gravity

In order to make manifest that teleparallel gravity is constructed over a nonlinear Riemann–Cartan geometry, we briefly review its fundamentals [1]. These fundamentals were originally devised to form a gauge theory for the Poincaré translations. By pinning down its Cartan geometric structure, it will be easy to understand how to allow for a different kind of kinematics in Section 4.

Because kinematics is governed by the Poincaré group, spacetime $\mathcal{M}$ is the base manifold of a principal $ISO(1,3)$ bundle Q, while the local Minkowski spaces are the fibers $\{M_x \mid x \in \mathcal{M}\}$ of the associated bundle $Q \times_{ISO(1,3)} M$. The $\mathfrak{iso}(1,3)$-valued connection $A_\mu dx^\mu$ determines how adjacent fibers compare under an infinitesimal displacement dx^μ.

In the absence of gravity the connection is flat, and spacetime can be identified with every local Minkowski space as follows. We adopt spacetime coordinates $x^\mu = \delta^\mu_a \xi^a$ for the Cartesian system ξ^a, while the section $\xi^a(x) = \delta^a_\mu x^\mu$ of $Q[M]$ defines the points of tangency between the local Minkowski spaces and spacetime. It follows that the vierbein is given trivially by $e^a{}_\mu = \delta^a_\mu$. After a general coordinate transformation $\delta^\mu_a \xi^a \mapsto x^\mu(\xi)$ the vierbein assumes the form $e^a{}_\mu = \partial_\mu \xi^a$. There yet remains the freedom to consider local $ISO(1,3)$ transformations on the tangent Minkowski spaces. Since elements of M_x transform as vectors under local Lorentz transformations $\xi^a \mapsto \Lambda^a{}_b\xi^b$, we provide the vierbein with a corresponding connection, i.e., $e^a{}_\mu = \partial_\mu\xi^a + A^a{}_{b\mu}\xi^b$. The connection is purely inertial, i.e., it takes the form $\Lambda^a{}_c\partial_\mu\Lambda_b{}^c$, so that no gravitational degrees of freedom are attributed to it. Lastly, the components of Lorentz vectors are invariant under local Poincaré translations $\xi^a \mapsto \xi^a + \epsilon^a$. Therefore, one includes a term in its definition

$$e^a{}_\mu = \partial_\mu\xi^a + A^a{}_{b\mu}\xi^b + A^a{}_\mu, \tag{9}$$

which transforms as $A^a{}_\mu \mapsto A^a{}_\mu - \partial_\mu\epsilon^a - A^a{}_{b\mu}\epsilon^b$. Note that the connection $A^a{}_{b\mu}$ is invariant under local Poincaré translations, hence

$$\omega^a{}_{b\mu} = A^a{}_{b\mu} \tag{10}$$

is a well-defined spin connection, being independent of the gauge ξ^a chosen.

The curvature and torsion then take the form

$$B^a{}_{b\mu\nu} = F^a{}_{b\mu\nu} \tag{11}$$

and

$$G^a{}_{\mu\nu} = \xi^b F^a{}_{b\mu\nu} + F^a{}_{\mu\nu}, \tag{12}$$