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Statistics of neutrinos and relativistic effective degrees of freedom in the early universe

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ABSTRACT

We study the effects of the presence of non-pure fermionic neutrinos on the relativistic effective degrees of freedom g_* in the early universe. The statistics of neutrinos is transformed from Fermi–Dirac (FD) to Bose–Einstein (BE) via Maxwell–Boltzmann (MB) statistics. The equilibrium energy density of pure bosonic neutrinos is larger than the energy density of pure fermionic neutrinos. One may expect that the relation $g_*^{\mathrm{FD}} < g_*^{\mathrm{MB}} < g_*^{\mathrm{BE}}$. We show that this relation is not always satisfied with degenerate neutrinos. We discuss briefly the cosmological consequences of this transformation for dark matter problem as well as the baryon–photon ratio in the universe.

1. Introduction

Neutrinos have spin 1/2 and it is natural that we understand the neutrinos obey purely Fermi–Dirac statistics on the analogy of the electrons [1]. However, properties of neutrinos are so different from other spin one half particles, e.g., tiny masses, large mixings, electrically neutral. The Pauli exclusion principle may be violated for neutrinos and neutrinos may possess mixed statistics [2–7].

Cosmology with non-pure fermionic neutrinos has been studied. For examples, Cucurull et al. [8] have shown that the ^{4}He abundance in the early universe can be used to evaluate the statistics of neutrinos. Dolgov et al. [9] have assumed that neutrinos obey Bose–Einstein statistics and discussed the cosmological and astrophysical consequences of this assumption for dark matter problem, spectra of the supernova neutrinos and ultra high energy cosmic rays. Also, Dolgov et al. have introduced the Fermi–Bose parameter and studied the effects of continuous transition from Fermi–Dirac to Bose–Einstein statistics of neutrinos on the effective number of neutrinos at big bang nucleosynthesis (BBN) [10]. A very rough estimation of the effective number of neutrinos as a function of the Fermi–Bose parameter, without careful consideration of the weak reactions of neutron–proton transformation $e^- + p \leftrightarrow n + \nu_e$ and the n/p freezing temperature, was also given in Ref. [11]. In these studies except Ref. [11], phenomenologies *at MeV scale (BBN scale)* are mainly discussed.

At GeV scale, the effects of continuous transition *from Fermi–Dirac to Maxwell–Boltzmann* statistics of neutrinos have been studied by the authors [11]. The equilibrium energy density of pure bosonic neutrinos is larger than the energy density of pure fermionic neutrinos by the factor 8/7 [12]. One may expect that the relativistic effective degrees of freedom in the radiation dominant era with neutrinos in Maxwell–Boltzmann statistics g_*^{MB} is larger than it with neutrinos in Fermi–Dirac statistics g_*^{FD} ($g_*^{\mathrm{FD}} < g_*^{\mathrm{MB}}$). In spite of this naive expectation, we have shown in our previous work [11] that the transition from pure Fermi–Dirac statistics to Maxwell–Boltzmann statistics of neutrinos decreases the relativistic effective degrees of freedom ($g_*^{\mathrm{MB}} < g_*^{\mathrm{FD}}$) if there are constant and large lepton asymmetries at GeV scale.

In this paper, we include the effects of continuous transition *from Bose–Einstein to Maxwell–Boltzmann* statistics of neutrinos at GeV scale on our discussion. The assumption of violation of the Pauli exclusion principle makes problems. We discuss these problems later. One may expect that the relation $g_*^{\mathrm{FD}} < g_*^{\mathrm{MB}} < g_*^{\mathrm{BE}}$ where g_*^{BE} denotes the relativistic effective degrees of freedom with pure bosonic neutrinos, however, we show that this relation is not always satisfied with degenerate neutrinos. For example, if there are strongly degenerate neutrinos, or equivalently there are large lepton asymmetries at GeV scale, $g_*^{\mathrm{BE}} < g_*^{\mathrm{MB}} < g_*^{\mathrm{FD}}$ is allowed. The large lepton asymmetries at MeV scale are almost excluded by the standard BBN cosmology [13,14]. However, the large lepton asymmetries at GeV scale are still compatible with current observations [15].

This paper is organized as follows. In Section 2, we discuss the problem of the violation of the Pauli exclusion principle. In Section 3, we transform the statistics of neutrinos from Fermi–Dirac

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to Bose–Einstein via Maxwell–Boltzmann statistics discretely and discuss the consequences of this transitions. In Section 4, the statistics of neutrinos is transformed continuously. In Section 5, we discuss briefly the cosmological consequences of this continuous transformation for dark matter problem and the baryon–photon ratio in the universe. Finally, Section 6 is devoted to a summary.

2. Violation of the Pauli exclusion principle

As we say in the introduction, the assumption of violation of the Pauli exclusion principle (PEP) makes problems. The scenario with PEP-violating particles is not a common idea in the literature. A discussion of why we would consider this scenario is needed for the readers.

Theoretical ideas of violation of PEP in non-standard types of statistics, such as parastatistics [16,17], "quons" with bilinear algebra [18–21], small PEP violation with trilinear algebra [3,22,23], have been studied (see also [24]). Ignatiev and Kuzmin have proved that either the upper limit on the occupancy number is just one (the pure Fermi–Dirac statistics) or there is no upper limit at all (such as the pure Bose–Einstein statistics as well as non-standard quon statistics) for an arbitrary system with a bilinear commutation relation [3], however, other ideas, such as non-standard statistics with trilinear algebra, are not excluded.

As Cucurull et al. have already addressed [8], the experimental evidence of the spin–statistics connection for ordinary matter in macroscopic samples is overwhelming. The specific heats of metals at low temperatures can only be explained if electrons in matter obey pure Fermi–Dirac statistics. The blackbody spectrum of radiation, such as cosmic microwave background radiation, is highly consistent with Bose–Einstein character of statistical ensembles of photons. However, neutrinos are not usually found in macroscopic samples, except hot supernova core [2], their statistical behaviour is difficult to be experimentally established. Thus, the Pauli exclusion principle may be violated for neutrinos and neutrinos may possess mixed statistics.

The two-neutrino double-beta decay experiments on ^{100}Mo to excited states of ^{100}Ru could provide the necessary experimental data depends on neutrino statistics [4,7]. The energy spectra of electrons show that the possibility of the presence of the pure bosonic neutrinos is almost excluded by double-beta decay experiments [9,25] (see also [5]). The decay rate for $0^+ \rightarrow 0^+$ transitions is suppressed for bosonic neutrinos compared to fermionic one. On the other hands, the decay rate for $0^+ \rightarrow 2^+$ transition is enhanced for bosonic neutrinos compared to the fermionic one [9,25]. The ratio

$$R(J^\pi) = \frac{T_{1/2}(J^\pi)}{T_{1/2}(0^+_{g.s.})}, \tag{1}$$

is useful to test the neutrino statistics [25], where $T_{1/2}(0^+_{g.s.})$ and $T_{1/2}(J^\pi)$ denote the half-life time for decay of ^{100}Mo to the ground state $0^+_{g.s.}$ and to the excited state J^π of ^{100}Ru, respectively. The theoretical predictions of the ratio for excited 0^+_1 state with pure fermionic neutrinos $R^{\mathrm{FD}}_{\mathrm{theo.}}(0^+_1)$ and with pure bosonic neutrinos $R^{\mathrm{BE}}_{\mathrm{theo.}}(0^+_1)$ are obtained as

$$R^{\mathrm{FD}}_{\mathrm{theo.}}(0^+_1) \simeq 61, \qquad R^{\mathrm{BE}}_{\mathrm{theo.}}(0^+_1) \simeq 73, \tag{2}$$

within the single state dominance (SSD) approach [25]. From the recent NEMO-3 data $T_{1/2}(0^+_1) = (7.5 \pm 0.6(stat) \pm 0.6(syst)) \times 10^{20}$ yr [7] with $T_{1/2}(0^+_{g.s.}) = (7.11 \pm 0.02(stat) \pm 0.54(syst)) \times 10^{18}$ yr [6], we obtain

$$R_{\mathrm{exp.}}(0^+_1) \simeq 82 - 132. \tag{3}$$

Table 1
Abbreviations.

Abbr.	Statistics	Θ_i	κ_i
FD	Fermi–Dirac	1	1
FB	Fermi–Boltzmann		$0 < \kappa_i \le 1$
MB	Maxwell–Boltzmann	0	0
BB	Bose–Boltzmann		$-1 \le \kappa_i < 0$
BE	Bose–Einstein	−1	−1

It seems that a bosonic neutrino fits the data slightly better, however, the associated experimental and theoretical uncertainties are too large to obtain the conclusions. In the case of 2^+_1 excited state, the theoretical predictions are obtained as [25]

$$R^{\mathrm{FD}}_{\mathrm{theo.}}(2^+_1) \simeq 2.5 \times 10^4, \qquad R^{\mathrm{BE}}_{\mathrm{theo.}}(2^+_1) \simeq 2.7 \times 10^2. \tag{4}$$

The observations of decay to the 2^+ excited state is more difficult. Therefore only a limit $T_{1/2}(2^+_1) > 2.5 \times 10^{21}$ yr has been reported [7]. Taking this limit, we obtain

$$R_{\mathrm{exp.}}(2^+_1) \gtrsim 3.4 \times 10^2. \tag{5}$$

It seems that the experimental result favours a fermionic neutrino, however, the uncertainties are too large to obtain the conclusions.

Although, the possibility of the presence of the pure bosonic neutrinos is almost excluded by the energy spectra of electrons on the double-beta decay [9], it does not mean that the neutrinos are pure fermions (a partly bosonic neutrinos is still allowed) [25]. Moreover, there is no strong experimental evidence that neutrinos obey pure Fermi–Dirac statistics [4]. In this paper, we put aside discussion of the particular mechanism of the PEP violation in the neutrino sector and we restrict ourselves to the phenomenological approach.

3. Discrete transition

3.1. Distribution function

In this section, we discuss the consequences of the discrete transition of the statistics of neutrinos from Fermi–Dirac to Bose–Einstein via Maxwell–Boltzmann distributions. As we mentioned in Section 2, the presence of the pure bosonic neutrinos is almost excluded by the double-beta decay experiments, however, we include the case of pure-bosonic neutrinos on our considerations to compare with the case of quasi-bosonic neutrinos in Section 4.

In the standard quantum statistics, the distribution function of particle species i is given by

$$f_i = \frac{g_i}{e^{(E_i - \mu_i)/T} + \Theta_i}, \tag{6}$$

where g_i, E_i, μ_i and T denote internal degrees of freedom, energy, chemical potential and temperature of particle i, respectively. The discrete parameter Θ_i takes only the following three values: $\Theta_i = 1, 0, -1$ for particles obeying Fermi–Dirac (FD), Maxwell–Boltzmann (MB) and Bose–Einstein (BE) statistics [12] (Table 1). If we treat Bose–Einstein condensate, $(2\pi)^3 n_0 \delta(\vec{p})$ should be added in Eq. (6) where n_0 and $\vec{p}$ denote the number density and momentum of bosons in the condensate [9,26].

The number density n_i, energy density ρ_i, pressure P_i and entropy density s_i of particle i are obtained as follows [12]:

$$n_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} E(E^2 - m_i^2)^{1/2} f_i dE, \tag{7}$$

$$\rho_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} E^2(E^2 - m_i^2)^{1/2} f_i dE, \tag{8}$$