Physics of the Dark Universe 4 (2014) 86–91

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Gravitational wave astronomy and cosmology

Scott A. Hughes

Department of Physics and MIT Kavli Institute, 77 Massachusetts Avenue, Cambridge, MA 02139, United States

ARTICLE INFO

Keywords:
Gravitational waves
Cosmology
Gravitation

ABSTRACT

The first direct observation of gravitational waves' action upon matter has recently been reported by the BICEP2 experiment. Advanced ground-based gravitational-wave detectors are being installed. They will soon be commissioned, and then begin searches for high-frequency gravitational waves at a sensitivity level that is widely expected to reach events involving compact objects like stellar mass black holes and neutron stars. Pulsar timing arrays continue to improve the bounds on gravitational waves at nanohertz frequencies, and may detect a signal on roughly the same timescale as ground-based detectors. The science case for space-based interferometers targeting millihertz sources is very strong. The decade of gravitational-wave discovery is poised to begin. In this writeup of a talk given at the 2013 TAUP conference, we will briefly review the physics of gravitational waves and gravitational-wave detectors, and then discuss the promise of these measurements for making cosmological measurements in the near future.

1. Introduction and overview

Although often introduced as a consequence of Einstein's theory of general relativity (GR), gravitational radiation is in fact necessary in any relativistic theory of gravity. These waves are simply the mechanism by which changes in gravity are causally communicated from a dynamical source to distant observers. In GR, the curvature of spacetime (which produces tidal gravitational forces) is the fundamental field characterizing gravity. Gravitational waves (GWs) are propagating waves of spacetime curvature, tidally stretching and squeezing as they radiate from their source into the universe.

Tidal fields are quadrupolar, so GWs typically arise from some source's bulk, quadrupolar dynamics. Consider a source whose mass and energy density are described by ρ. Choosing the origin of our coordinates at the source's center of mass, its quadrupole moment is given by

$$Q_{ij} = \int \rho \left(x_i x_j - \frac{1}{3}\delta_{ij} r^2 \right) dV, \tag{1}$$

where the integral is taken over the source. The gravitational-wave potential, h_{ij}, comes from the second time derivative of Q_{ij}:

$$h_{ij} = \frac{2G}{c^4}\frac{1}{r}\frac{d^2 Q_{ij}}{dt^2}, \tag{2}$$

where r is distance from the source to the observer. The magnitude of a typical component of h_{ij} is

$$h \approx \frac{G}{c^4}\frac{mv^2}{r}, \tag{3}$$

where v is the typical speed associated with the source's quadrupolar dynamics, and m is the mass that participates in those dynamics. Notice the combination of constants appearing here,

$$\frac{G}{c^4} = 8.27 \times 10^{-50}\ \mathrm{gm}^{-1}\mathrm{cm}\left(\frac{\mathrm{cm}}{\mathrm{s}}\right)^{-2}. \tag{4}$$

This is rather small, reflecting the fact that gravity is the weakest of the fundamental forces. To overcome it, one must typically have large masses moving very quickly. A short-period binary in which each member is a compact object (white dwarf, neutron star, or black hole) is a perfect example of a strong quadrupolar radiator. For many of the sources we discuss, m is of order solar masses (or even millions of solar masses), and v is a substantial fraction of the speed of light.

(In addition to quadrupole dynamics, there is one other well-known mechanism for producing GWs: the amplification of primordial ground-state fluctuations by rapid cosmic expansion. We will briefly discuss this way of producing GWs in Section 2.4.)

The GWs a source emits backreact upon it, which appears as a loss of energy and angular momentum. The "quadrupole formula" predicts that a system with a time changing quadrupole moment will lose energy to GWs according to

$$\frac{dE}{dt} = -\frac{1}{5}\frac{G}{c^5}\sum_{ij}\frac{d^3 Q_{ij}}{dt^3}\frac{d^3 Q_{ij}}{dt^3}. \tag{5}$$

This loss of energy from, for example, a binary star system will appear as a secular decrease in the binary's orbital period—orbital

E-mail address: sahughes@mit.edu.

http://dx.doi.org/10.1016/j.dark.2014.10.003

energy is lost to GWs and the stars fall closer together. This effect was first seen in the first known binary neutron star system, PSR 1913 + 16 (the famed "Hulse–Taylor" pulsar) [1]. In this system, each neutron star has a mass slightly over $1.4\,M_\odot$, and they orbit each other in less than 8 h. The period has been observed to decrease by about 40 s over a baseline of nearly 40 years of observation. Similar period evolutions have now been measured in about 10 galactic binaries containing pulsars [2], and has even been seen in optical measurements of a close white dwarf binary system [3].

From these "indirect" detections, a major goal now is to directly detect GWs. Except at the longest wavelengths (where direct detection has recently been reported), almost all measurement schemes use the fact that a GW causes oscillations in the time of flight of a light signal; the basic idea was sketched by Bondi in 1957 [4]. Imagine an emitter located at $x = x_e$ that generates a series of very regular pulses, and a sensor at $x = x_s$. Ignoring the nearly static contribution of local gravitational fields (e.g., from the Earth and our solar system), the spacetime metric through which the light pulses travel can be written as

$$ds^2 = -c^2dt^2 + [1 + h(t)]\,dx^2. \tag{6}$$

Light moves along a null trajectory for which $ds^2 = 0$, which means that the speed of light with respect to these coordinates is (bearing in mind that $h \ll 1$)

$$\frac{dx}{dt} = c\left[1 - \frac{1}{2}h(t)\right]. \tag{7}$$

The time it takes light to travel from the emitter to the receiver is

$$\Delta T = \int_{x_e}^{x_s} \frac{dx}{dx/dt} = \frac{x_s - x_e}{c} + \frac{1}{2c}\int_{x_e}^{x_s} h(t)\,dx. \tag{8}$$

The gravitational wave thus enters as an oscillation in the arrival time of pulses. If the emitter is regular enough to be a precise clock, one may measure the GW by measuring this oscillation.

Before rushing out to build our detector, we should estimate how strong the gravitational waves we seek are. We use the formula for h given above, substituting fiducial values for the physical parameters that are likely to characterize the sources we aim to measure:

$$\begin{aligned} h &\approx \frac{G}{c^4}\frac{mv^2}{r} \\ &\approx 10^{-22} \times \left(\frac{200\,\text{Mpc}}{r}\right) \times \left(\frac{M}{3M_\odot}\right) \times \left(\frac{v}{0.3c}\right)^2 \\ &\approx 10^{-20} \times \left(\frac{6\,\text{Gpc}}{r}\right) \times \left(\frac{M}{10^6\,M_\odot}\right) \times \left(\frac{v}{0.1c}\right)^2. \end{aligned} \tag{9}$$

The first set of numbers characterizes stellar mass sources that are targets for ground-based high-frequency detectors, discussed in Section 2.1; the second characterizes massive black holes that are targets of space-based low-frequency detectors discussed in Section 2.2, and (at somewhat higher M, lower v, and smaller r) of pulsar timing arrays discussed in Section 2.3.

The numbers for h are *tiny*. Measuring timing oscillations at this level of precision might seem crazy. However, there is no issue of principle that prevents us from measuring effects at this level; the real challenge is to ensure that noise does not obscure the signal we hope to measure. Recall that a gravitational wave acts as a tidal force. The tide per unit mass for a GW of amplitude h and frequency ω is $R \simeq \omega^2 h$. Considering a light source and sensor separated by distance L, this means that we must control against stray forces on our test mass m of magnitude

$$\begin{aligned} F \simeq mL\omega^2 h &\simeq 6\text{ piconewtons}\left(\frac{m}{40\,\text{kg}}\right)\left(\frac{f}{100\,\text{Hz}}\right)^2 \\ &\times \left(\frac{L}{4000\,\text{m}}\right)\left(\frac{h}{10^{-22}}\right). \end{aligned} \tag{10}$$

(These fiducial parameters correspond to the LIGO observatories.) Six piconewtons is small, but it is well within our reach to isolate against forces of this magnitude—this is roughly the weight of a single animal cell. Though challenging, measuring a GW of $h \sim 10^{-22}$ is within our grasp.

In the remainder of this article, we discuss some of the science of GWs. We break up our discussion by frequency band. We begin with the *high frequency* band, with wave frequencies ranging from Hz to kHz, which are targeted by ground-based interferometers; then move to *low frequency*, waves with periods of minutes to hours, which are targets of space-based interferometers; then *very low frequency*, waves with periods of order months to years, which are targets of pulsar timing arrays; and finally conclude with *ultra low frequency*, with wavelengths comparable to the size of the universe.

2. The spectrum of gravitational waves

2.1. High frequency

The high-frequency band of roughly 1–1000 Hz is targeted by ground-based laser interferometers. The lower end of this band is set by gravitational coupling to local seismic disturbances, which can never be isolated against [5]; the upper end is set by the fact that 1 kHz is roughly the highest frequency that one expects from astrophysical strong GW sources. In laser interferometry, the laser's very stable frequency serves as the clock for the measurement procedure sketched in Section 1. GWs are detected by their action on light propagating between widely separated (hundreds to thousands of meters) test masses.

Several facilities around the world are involved in the search for GWs. Some of these facilities are presently offline as they undergo upgrades to "advanced" sensitivity, but will begin active GW searches again in about two years. There is very close collaboration among the facilities' research groups; combining data from multiple observatories greatly increases the ability to discriminate against noise and to insure detection. The most sensitive instruments in the worldwide network are associated with the Laser Interferometer Gravitational-wave Observatory, or LIGO. LIGO has a pair of four kilometer, L-shaped interferometers located in Hanford, Washington and Livingston, Louisiana. Closely associated with LIGO is GEO600, a 600 m interferometer near Hannover, Germany. Because of its shorter arms, GEO cannot achieve the same sensitivity as the LIGO detectors. However, it has been used as a testbed for advanced interferometry techniques, which has allowed it to maintain its role as an important part of the worldwide detector network. Completing the present network is Virgo, a 3 km interferometer located in Pisa, Italy, and operated by a French–Italian collaboration. Its sensitivity is fairly close to that of the LIGO instruments. Discussion of recent performance and upgrade plans for these three instruments can be found here [6].

A source of mass M and size R has a natural GW frequency of $f \sim (1/2\pi)\sqrt{GM/R^3}$. A compact source has size $R \sim$ several $\times$ GM/c^2. For such sources, the natural GW frequency is in the high-frequency band if $M \sim 1$–$100\,M_\odot$. For this reason, the high-frequency band largely targets objects like neutron stars and black holes. One of the most important sources in this band is the coalescence of binary neutron star systems—essentially, the last several minutes of systems like the Hulse–Taylor binary pulsar. Binaries containing black holes may also be important sources, though our poorer understanding of the formation of compact binaries with black holes make their rates substantially less certain.

As mentioned above, the LIGO and Virgo instruments are presently undergoing an upgrade to "advanced" sensitivity, which will give them a reach to binary neutron star inspiral of about 200