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An estimate of Λ in resummed quantum gravity in the context of asymptotic safety☆

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ABSTRACT

We show that, by using recently developed exact resummation techniques based on the extension of the methods of Yennie, Frautschi and Suura to Feynman's formulation of Einstein's theory, we get quantum field theoretic descriptions for the UV fixed-point behaviors of the dimensionless gravitational and cosmological constants postulated by Weinberg. Connecting our work to the attendant phenomenological asymptotic safety analysis of Planck scale cosmology by Bonanno and Reuter, we estimate the value of the cosmological constant Λ. We find the encouraging estimate $\rho_\Lambda \equiv \frac{\Lambda}{8\pi G_N} \simeq (2.4 \times 10^{-3}\ \text{eV})^4$. While this numerical value is close to recent experimental observations, we caution the reader that the estimate involves a number of model parameters that still possess significant levels of uncertainty, such as the value of the transition time between the Planck scale cosmology era and the Friedmann–Robertson–Walker radiation dominated era, where our current understanding allows for at least two orders of magnitude in its uncertainty and this would change our estimate of ρ_Λ by at least four orders of magnitude. We discuss such theoretical uncertainties as well. We show why GUT and EW scale vacuum energies from spontaneous symmetry breaking are suppressed in our approach to the estimation of ρ_Λ. As a bonus, we show how our estimate constrains susy GUTS.

1. Introduction

In Ref. [1], Weinberg suggested that the general theory of relativity may have a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit, so that it would be asymptotically safe with an S-matrix that depends on only a finite number of observable parameters. In Refs. [2–7], strong evidence has been calculated using Wilsonian [8] field-space exact renormalization group methods to support Weinberg's asymptotic safety hypothesis for the Einstein-Hilbert theory. As we review briefly below, in a parallel but independent development [9–18], we have shown [19] that the extension of the amplitude-based, exact resummation theory of Refs. [20,21] to the Einstein–Hilbert theory leads to UV-fixed-point behavior for the dimensionless gravitational and cosmological constants, but with the added bonus that the resummed theory is actually UV finite when expanded in the resummed propagators and vertices to any finite order in the respective improved loop expansion. We have called the resummed theory resummed quantum gravity. More recently, more evidence for Weinberg's asymptotic safety behavior has been calculated using causal dynamical triangulated lattice methods in Ref. [22].[1] At this point, there is no known inconsistency between our analysis and those of the Refs. [2–7,22].

We need to stress that the results in Refs. [2–7], while impressive, involve cut-offs which remain in the results to varying degrees even for products such as that for the UV limits of the dimensionless gravitational and cosmological constants. In addition, the results in Refs. [2–7] retain some mild dependence on gauge parameters, again even for the product of the UV limits of the dimensionless gravitational and cosmological constants. Accordingly, henceforward, we refer to the approach in Refs. [2–7] as the 'phenomenological' asymptotic safety approach. What can be said is that dependencies are mild enough that the existence of the non-Gaussian UV fixed point found in these references is probably a physical result. But, until a rigorously cut-off independent and gauge invariant calculation corroborates these results, we cannot consider them final. Our approach offers such a calculation, as our results are both gauge invariant and cut-off independent. The results from Ref. [22], involving, as they most certainly do, lattice constant-type artifact issues, are also only an indication of what the true continuum limit might realize – they too need to be corroborated by a rigorous calculation without the issues of finite size and other possible lattice artifacts to be considered final. Again, our

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[1] We also note that the model in Ref. [23] realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the UV-fixed point but it does so at the expense of violating Lorentz invariance.

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approach offers an answer to these issues. The stage is therefore prepared for us to try to make contact with experiment, as such contact is the ultimate purpose of theoretical physics.

Toward this end, we note that, in Refs. [24,25], it has been argued that the attendant phenomenological asymptotic safety approach in Refs. [2–7] to quantum gravity may indeed provide a realization[2] of the successful inflationary model [27,28] of cosmology without the need of the as yet unseen inflaton scalar field: the attendant UV fixed point solution allows one to develop Planck scale cosmology that joins smoothly onto the standard Friedmann–Walker–Robertson classical descriptions so that then one arrives at a quantum mechanical solution to the horizon, flatness, entropy and scale free spectrum problems. In Ref. [19], we have shown that, in the new resummed theory [9–18] of quantum gravity, we recover the properties as used in Refs. [24,25] for the UV fixed point of quantum gravity with the added results that we get “first principles” predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants in their analysis. In what follows here, we carry the analysis one step further and arrive at an estimate for the observed cosmological constant Λ in the context of the Planck scale cosmology of Refs. [24,25]. We comment on the reliability of the result as well, as it will be seen already to be relatively close to the observed value [29,30]. While we obviously do not want to overdo the closeness to the experimental value, we do want to argue that this again gives, at the least, some more credibility to the new resummed theory as well as to the methods in Refs. [2–7,22]. More reflections on the attendant implications of the latter credibility in the search for an experimentally testable union of the original ideas of Bohr and Einstein will be taken up elsewhere [31].

The discussion is organized as follows. We start by recapitulating the Planck scale cosmology presented phenomenologically in Refs. [24,25]. This is done in the next section. We then review our results in Ref. [19] for the dimensionless gravitational and cosmological constants at the UV fixed point. In the course of this latter review, which is done in Section 3 , we give a new proof of the UV finiteness of the resummed quantum gravity theory for the sake of completeness. In Section 4 , we then combine the Planck scale cosmology scenario in Refs. [24,25] with our results to estimate the observed value of the cosmological constant Λ. The appendices contain relevant technical details.

2. Planck scale cosmology

More precisely, we recall the Einstein–Hilbert theory

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda), \tag{1}$$

where R is the curvature scalar, g is the determinant of the metric of space-time $g_{\mu\nu}$, Λ is the cosmological constant and $\kappa = \sqrt{8\pi G_N}$ for Newton's constant G_N . Using the phenomenological exact renormalization group for the Wilsonian [8] coarse grained effective average action in field space, the authors in Refs. [24,25] have argued that the attendant running Newton constant $G_N(k)$ and running cosmological constant $\Lambda(k)$ approach UV fixed points as k goes to infinity in the deep Euclidean regime in the sense that $k^2\, G_N(k) \to g_*$, $\Lambda(k) \to \lambda_*\, k^2$ for $k \to \infty$ in the Euclidean regime.

The contact with cosmology then proceeds as follows. Using a phenomenological connection between the momentum scale k characterizing the coarseness of the Wilsonian graininess of the average effective action and the cosmological time t, the authors in Refs. [24,25] show that the standard cosmological equations admit of the following extension:

$$\begin{aligned}
\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} &= \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho\\
\dot\rho + 3(1+\omega)\frac{\dot a}{a}\rho &= 0\\
\dot\Lambda + 8\pi\rho\dot G_N &= 0\\
G_N(t) &= G_N(k(t))\\
\Lambda(t) &= \Lambda(k(t))
\end{aligned} \tag{2}$$

in a standard notation for the density ρ and scale factor $a(t)$ with the Robertson–Walker metric representation as

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right) \tag{3}$$

so that $K = 0,\ 1,\ -1$ correspond respectively to flat, spherical and pseudo-spherical 3-spaces for constant time t. Here, the equation of state is taken as

$$p(t) = \omega\rho(t), \tag{4}$$

where p is the pressure. In Refs. [24,25] the functional relationship between the respective momentum scale k and the cosmological time t is determined phenomenologically via

$$k(t) = \frac{\xi}{t} \tag{5}$$

for some positive constant ξ determined from requirements on physically observable predictions.

Using the UV fixed points as discussed above for $k^2 G_N(k) \equiv g_*$ and $\Lambda(k)/k^2 \equiv \lambda_*$ obtained from their phenomenological, exact renormalization group (asymptotic safety) analysis, the authors in Refs. [24,25] show that the system in (2) admits, for $K = 0$, a solution in the Planck regime where $0 \le t \le t_{\rm class}$, with $t_{\rm class}$ a “few” times the Planck time t_{Pl}, which joins smoothly onto a solution in the classical regime, $t > t_{\rm class}$, which coincides with standard Friedmann–Robertson–Walker phenomenology but with the horizon, flatness, scale free Harrison–Zeldovich spectrum, and entropy[3] problems all solved purely by Planck scale quantum physics.

While the dependencies of the fixed-point results g_*, λ_* on the cut-offs used in the Wilsonian coarse-graining procedure, for example, make the phenomenological nature of the analyses in Refs. [24,25] manifest, we note that the key properties of g_*, λ_* used for these analyses are that the two UV limits are both positive and that the product $g_*\ \lambda_*$ is only mildly cut-off/threshold function dependent. Here, we review the predictions in Ref. [19] for these UV limits as implied by resummed quantum gravity theory as presented in [9–18] and show how to use them to predict the current value of Λ. In view of the lack of familiarity of the resummed quantum gravity theory, we start the next section with a review of its basic principles in the interest of making the discussion self-contained.

3. g_* and λ_* in resummed quantum gravity

We start with the prediction for g_*, which we already presented in Refs. [9–19]. Given that the theory we use is not very familiar, we recapitulate the main steps in the calculation in the interest of completeness.

More specifically, as the graviton couples to a an elementary particle in the infrared regime which we shall resum independently of the particle's spin, we may use a scalar field to develop the required calculational framework. The extension to spinning particles will then be straightforward. Thus, we start with the Lagrangian density for the

[2] The attendant choice of the scale $k \sim 1/t$ used in Refs. [24,25] was also proposed in Ref. [26].

[3] Here, we should note that, to solve the entropy problem, the authors in Ref. [25] retain the general form of the requirement from Bianchi's identity so that the second and third relations in (2) are combined to $\dot\rho + 3(1+\omega)\frac{\dot a}{a}\rho = -\frac{\dot\Lambda + 8\pi\rho\dot G_N}{8\pi G_N}$; we discuss this in more detail in Section 4.