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H, G_1, G_2 photometric phase function extended to low-accuracy data

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ABSTRACT

We introduce a constrained nonlinear least-squares algorithm to be used in estimating the parameters in the H, G_1, G_2 phase function. As the algorithm works directly in the magnitude space, it will surpass the possible bias problem that may be present in the existing H, G_1, G_2 fit procedure when applied to low-accuracy observations with large magnitude variations. With constraints on the photometric phase-curve shape parameters G_1 and G_2, it guarantees a physically reasonable phase-curve estimate. With a new data set of 93 asteroids, we re-assess the two-parameter version of the H, G_1, G_2 function. Finally, we introduce a one-parameter version of the phase function that can give a suggestion of the asteroids taxonomic group based only on its phase curve. A statistical model selection procedure is presented that can automatically select between the different versions of the photometric phase functions. An online tool that implements these algorithms is introduced.

1. Introduction

The reliable estimation of the absolute magnitude H for an asteroid from the photometric observations is extremely important. The absolute magnitude (the apparent magnitude at 1 au from the Sun and the Earth, observed in the backscattering direction) relates the brightness of the asteroid to its size, if the albedo of the target is known. Furthermore, the shape of the photometric phase curve (i.e., magnitude as a function of the phase angle) can serve as a proxy for the taxonomic type of the asteroid in cases when spectral information is not available (see, e.g., Oszkiewicz et al., 2012; Shevchenko et al., 2016, and Section 3.5).

The International Astronomical Union (IAU) adopted the so-called H, G photometric phase function in 1989 to be used when estimating the shape of the photometric phase curve and the absolute magnitude H (Bowell et al., 1989). In 2012, the IAU adopted the so-called H, G_1, G_2 function (Muinonen et al., 2010). The H, G_1, G_2 function improved especially the backscattering behavior of the curve with high- and low-albedo asteroids.

Both the H,G and the H, G_1, G_2 phase functions can be applied to targets with multiple high-quality observations. If the number of observations is small, or their accuracy is low, problems may arise. The most apparent problem is that, especially, the parameter G or the parameters G_1, G_2 might be poorly estimated. The solution with the H, G has been to fix value of G to a constant value and estimate only the H. The H, G_1, G_2 offers an improved solution with a two-parameter H, G_{12} function. Nevertheless, even the H, G_{12} function fit can be nonreliable with very small data sets, and procedures of using a fixed G_{12} values have been applied.

We offer a solution that we believe to improve the current situation with the photometric fits with a small number of low-accuracy observations. After a short definition of the H, G_1, G_2 phase function in Section 2, we present a constrained nonlinear least-squares method for fitting the H, G_1, G_2 function that can improve the possible bias with low-accuracy data (Section 3). Then, we revisit the two-parameter phase function with new data in Section 3.4 and offer a new version of that, the H, G^*_{12} phase function. In Section 3.5 we asses the problem with fixed G or G_1, G_2 parameters by introducing one-parameter models that relate to five taxonomic asteroid groups. We tie all the models with three, two or one parameter together in Section 3.6 by introducing a statistical model selection procedure to select the best version to be used with a particular data set. We have developed an online tool that implements the algorithms (Section 4), and show one application example with the near-Earth asteroid (144411) 2004 EW9 in Section 4.1.

2. The H, G_1, G_2 phase function

The H, G_1, G_2 photometric phase function is already extensively presented in Muinonen et al. (2010), thus we recall here only a short summary of the key concepts. The data to be modeled consists of triplets $(\alpha_i, V(\alpha_i), \sigma_i)$, where α is the phase angle between the Sun and the observer, as seen from the target. $V(\alpha)$ is the reduced

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Table 1
H, G_1, G_2 basis functions $\Phi_{1,2,3}(\alpha)$. For details about the splines $\xi_{1,2,3}(\alpha)$, see Appendix A, and for the tabulated basis function values, Appendix B.

Range (deg)	$\Phi_1(\alpha)$	Range (deg)	$\Phi_2(\alpha)$	Range (deg)	$\Phi_3(\alpha)$
0–7.5	$1-\frac{6}{\pi}\alpha$	0–7.5	$1-\frac{9}{5\pi}\alpha$	0–30	$\xi_3(\alpha)$
7.5–150	$\xi_1(\alpha)$	7.5–150	$\xi_2(\alpha)$	30–150	0

observed magnitude at phase angle α, and σ is the error (standard deviation) of the observation. Note that σ is always included in the H, G_1, G_2. If it is not given in the data, an implicit value of 0.03 mag is used. There is also an implicit assumption in the model that the error distribution in the reduced magnitude value is symmetric.

In the (reduced) magnitude value space, the H, G_1, G_2 model is of form

$$V(\alpha) = H - 2.5\log_{10}[G_1\Phi_1(\alpha) + G_2\Phi_2(\alpha) + (1 - G_1 - G_2)\Phi_3(\alpha)], \quad (1)$$

where H, G_1, and G_2 are the parameters of the model, and Φ_i are the basis functions. The basis functions are composite functions consisting of linear parts (in $\Phi_{1,2}$), constant part (in Φ_3), and parts defined by cubic splines $\xi(\alpha)$ (see Appendix A for details of the spline implementation). The model is valid from $\alpha = 0°$ to $150°$, and the basis functions are given in Table 1. For convenience, we also give the tabulated values of the basis functions in Appendix B.

In the original H, G_1, G_2 the fit between the data and the model is not done in the magnitude values, but in flux values. The magnitudes V are converted to flux F with a nonlinear relation $F = 10^{-0.4V}$. After the conversion, the model in Eq. (1) can be written as

$$F(\alpha) = a_1\Phi_1(\alpha) + a_2\Phi_2(\alpha) + (1 - a_1 - a_2)\Phi_3(\alpha) \quad (2)$$

with relations

$$H = -2.5\log_{10}(a_1 + a_2 + a_3), \quad G_1 = \frac{a_1}{a_1 + a_2 + a_3}, \quad G_2 = \frac{a_2}{a_1 + a_2 + a_3}. \quad (3)$$

The error in the magnitude space, σ, converts into the flux space, σ_F, depending also on the corresponding magnitude value V, by

$$\sigma_F = 10^{-0.4V}(10^{0.4\sigma} - 1). \quad (4)$$

Finally, the H, G_1, G_2 function in the flux space (Eq. (2)) is fitted to data (α_i, F_i) using the linear least-squares method with the weights $1/\sigma_{F,i}^2$, and the parameter values in the magnitude space are received by applying Eq. (3) to the estimates $(\widehat{a_1}, \widehat{a_2}, \widehat{a_3})$.

The errors for the parameter estimates $(\widehat{H}, \widehat{G_1}, \widehat{G_2})$, as well as other quantities derived from the fit, such as the photometric phase coefficient $k = -(1/(5\pi))(30G_1 + 9G_2)/(G_1 + G_2)$, are estimated using Monte Carlo simulation with the H, G_1, G_2 function. It is assumed that the linear estimates $(\widehat{a_1}, \widehat{a_2}, \widehat{a_3})$ follow the three-dimensional normal distribution. The covariance matrix of the distribution is $\mathbf{\Sigma} = (\mathbf{X}'\mathbf{X})^{-1}$, where $\mathbf{X}$ is the model matrix with the weighted values of the base functions at observed phase angles, $[\mathbf{X}]_{ij} = \Phi_j(\alpha_i)/\sigma_{F,i}^2$.

As m values $(a_1^{(k)}, a_2^{(k)}, a_3^{(k)})$, $k = 1, \ldots, m$ are simulated, Eq. (3) can be used to receive simulated sample $(H^{(k)}, G_1^{(k)}, G_2^{(k)})$. The error for $(\widehat{H}, \widehat{G_1}, \widehat{G_2})$ can then be assessed by computing standard deviation or quantiles from that simulated sample. Using quantiles for error estimation is preferred since the distribution for the nonlinear parameters can be non-symmetric.

3. H, G_1, G_2 fits with low-accuracy data

First of all, we like to stress that the original H, G_1, G_2 photometric phase function described in Section 2 works perfectly well in most cases. Usually, if we have 'enough' data (say, more than 5–10 observations) with modest variation ($\sigma \lesssim 0.05$ mag) and with enough spread in the phase angles, there are no problems. However, there are issues related to sparse data with large variations where an alternative approach in the way the H, G_1, G_2 function is fitted to the data can be fruitful. In what follows, we improve the usability of the H, G_1, G_2 model in these problematic cases, revisit the simplified versions of the model with fewer parameters, and suggest an automated method of selecting between the full model and models with fewer parameters.

3.1. Constraining the fit

In its original form the parameter values, either the flux space parameters (a_1, a_2, a_3) or the magnitude space parameters (H, G_1, G_2), are not constrained in any way. However, there are physical arguments to do so. Firstly, it is generally clear that the flux from the object should decrease when the phase angle increases, since the projected illuminated area of the object, as seen by the observer, is decreasing. Secondly, all the scattering processes that we can identify (i.e., coherent backscattering, shadow-hiding) are amplifying the decrease of flux when moving from exact backscattering. So, we can safely state that the flux space model $F(\alpha)$ in Eq. (2) should have a negative first derivative with all α. Thus, the magnitude space model $V(\alpha)$ in Eq. (1) should always have a positive first derivative.

The requirement of $F'(\alpha) \le 0$ is possible to fulfill by searching (numerically in the case of splines) conditions for (a_1, a_2, a_3). Unfortunately, since there are three composite basis functions, the conditions become quite complicated. To simplify the situation, we can take into account a mathematical or semi-physical requirement that the basis functions $\Phi_{1,2,3}$ in the flux space should have positive coefficients $a_{1,2,3}$. The functions Φ_1 and Φ_2 should bracket the photometric slope behavior of an asteroid (Muinonen et al., 2010, Section 3.2), thus it is natural that $0 \le a_1, a_2$. The basis function Φ_3 is used to introduce backscattering enhancement to the phase function, and therefore is natural to assume that $0 \le a_3$. With the abovementioned conditions, the first derivative of the flux model is always negative.

If we think about the magnitude space parameters (H, G_1, G_2), we can derive conditions for G_1 and G_2 based on the ones for the flux space parameters. From relations in Eq. (3), we can directly see that we can require

$$0 \le G_1, G_2, \quad 1 - G_1 - G_2 \le 1. \quad (5)$$

Introducing these conditions will require changing the linear least-squares method for fitting the model with the data into a general (nonlinear) constrained least-squares method.

3.2. Bias in the magnitude space with the linear fit in the flux space

The parameter of the greatest importance with the H, G_1, G_2 model is the predicted absolute magnitude H of the target. Observations are generally given in magnitudes and not in flux, and data is visualized in phase-magnitude plots. As mentioned earlier, the original H, G_1, G_2 model implicitly assumes symmetric distribution of errors in magnitude values. All these support the general idea that the fit should be unbiased in the magnitude space. Therefore, there is some controversy, at least in theory, in the fact that the function is defined to be fitted in the flux space. Since the transformation between the magnitude and the flux, $F = 10^{-0.4V}$, is nonlinear, the fit cannot be (strictly) unbiased in both spaces, and the error distribution cannot be (strictly) symmetric in both spaces. As the least-squares fit is done in the flux space, the model cannot be unbiased in the magnitude space. As the error distribution is