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Asteroid lightcurve inversion with Lommel–Seeliger ellipsoids

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ABSTRACT

We derive initial rotation, shape, and scattering properties for asteroids from sparse and dense photometry based on the so-called Lommel–Seeliger ellipsoid (LS ellipsoid). Due to the analytical disk-integrated brightness, the LS ellipsoid allows for fast rotation-period, pole-orientation, and shape analyses, as well as efficient Markov-chain Monte Carlo solutions (MCMC). We apply the methods to simulated sparse Gaia photometry, as well as to ground-based photometry composed of dense lightcurves. For a specific Gaia simulation, we make use of a numerical reflection coefficient developed for particulate surfaces, and utilize the LS ellipsoid in the inversion of the simulated data. We conclude that, in a majority of cases, initial LS ellipsoid retrieval of the parameters is satisfactory. Finally, we formulate a single-scattering phase function that, for a spherical asteroid, results in the H, G_1, G_2 photometric phase function.

1. Introduction

Asteroid rotation periods, pole orientations, and three-dimensional shapes are derived from photometric lightcurves observed in a number of apparitions with varying illumination and observation geometries (e.g., Kaasalainen et al., 2001; Torppa et al., 2008; Durech et al., 2009). It is customary to estimate the rotation period with a regular initial shape model and a small number of trial pole orientations. Once the period is available, the pole orientation and shape can be refined with a general convex shape model. We focus on the initial inverse problem exhibited by sparse photometry consisting of isolated observations over time intervals of several rotational periods or a number of dense lightcurves observed in a limited set of viewing geometries.

The morphology of asteroid lightcurves is affected by a variety of macroscopic and microscopic properties of the surface. The overall shape plays a major role and can be responsible for significant changes in the lightcurve of the same object observed at different epochs in a variety of observing conditions (aspect and phase angles). This was already mentioned by Cellino et al. (1989), in which an eight-octant model for asteroid shapes (the so-called "Cellinoid shape model") was first introduced. Some recent analyses (Lu and Ip, 2015) suggest that this shape model can be conveniently applied to the problem of inversion of asteroid lightcurves.

The inversion of asteroid photometric data can become even more difficult when one does not have at disposal full lightcurves, but only sparse photometric "snapshots" of an object taken at different epochs. This is exactly the problem that must be faced in the case of the inversion of Gaia photometric data, the average number of Gaia measurements being of the order of 70 per object. In the case of Gaia, there is the additional complication of the large number of objects to be processed (of the order of 10^5), and the challenge of the limited computing time available to accomplish this task in the framework of the Gaia data reduction activities. The solution that has been found is the application of a genetic algorithm that assumes the bodies to have regular triaxial ellipsoid shapes and finds the best combination of axial ratios, rotation period, and pole orientation that produces the best fit to the data. In so doing, also a linear variation of magnitude as a function of phase angle is assumed to take place (Cellino et al., 2009).

The results of the ellipsoid approach have been found to be satisfactory according to many numerical simulations. The most recent analysis has been published by Santana-Ros et al. (2015), who performed several sets of simulations for 10,000 asteroids having different periods, poles, and shapes. The effects of different photometric errors affecting the data was also simulated. The

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results confirmed the reliability of the inversion algorithm, and showed that, as expected, the obtained results are worse when inverting asteroids having quasi-spherical shapes and/or pole-on illumination and observation geometries. The latter effect is caused by the intrinsically small photometric variations occurring in these cases. Santana-Ros et al. also analyzed the effect of having at disposal, in addition to sparse Gaia data, a full lightcurve obtained from ancillary ground-based observations. The results suggest that it should be possible in this way to reduce the number of wrong solutions for asteroids having less than 50 photometric data points.

We assess the initial derivation of the rotational period and pole orientation with the help of the Lommel–Seeliger ellipsoid (LS ellipsoid), a triaxial ellipsoid with a Lommel–Seeliger surface reflection coefficient. The disk-integrated photometric brightness for the LS ellipsoid is available analytically (Muinonen and Lumme, 2015), warranting efficient computation of lightcurves and fast inverse methods. For earlier application of triaxial ellipsoids to asteroid lightcurve analyses, we refer the reader, for example, to the works by Drummond et al. (1988) and Magnusson et al. (1989).

With modern computers and the LS ellipsoid, the rotation period, pole orientation, and ellipsoidal shape can be derived simultaneously, e.g., by using a genetic algorithm (as described above and in Cellino et al., 2015). Here we proceed systematically as follows. First, the rotation period is scanned across its relevant range with a resolution $P_0^2/2T$ given by a tentative period estimate P_0 and the time interval T spanned by the photometric data. This is typically carried out for a small number of different pole orientations distributed, as uniformly as possible, on the unit sphere. For each rotation period and pole orientation, the pole orientation itself, rotational phase, and axial ratios (also, optionally, the scattering parameters) are optimized with the help of the Nelder–Mead downhill simplex method. Although the shape optimization can suffer from getting stuck in local minima, overall, the rotation period is reliably obtained by the initial scanning.

Second, for the rotation period obtained, the pole orientation can be mapped with a high resolution pertaining to a few degrees on the unit sphere, with downhill simplex optimization for the rotation period, rotational phase, and axial ratios (again, optionally also the scattering parameters) in the case of each trial pole orientation. Third, after mapping the pole orientation, the regimes of minima are evident, and analyses can be focused on each of the regimes separately. Now all the parameters can be optimized to obtain the best single fit to the data.

Next, in order to allow for an efficient Markov-chain Monte Carlo analysis (MCMC) in the proximity of the best-fit solution, we generate virtual observations, by adding random noise to the observations, and repeat the optimization for the parameters by using the virtual data (see Wang et al., 2015a; Muinonen et al., 2012). The differences of what we call the virtual least-squares parameters can then be utilized as proposals in the MCMC sampling of the parameters. At the end, we obtain a description of the probability density function for the period, pole, rotational phase, and ellipsoid axial ratios (also, optionally, the scattering parameters) in the neighborhood of the best-fit parameters. The MCMC analysis can be repeated for each different solution regime separately and mutual weights of the separate regimes can be derived. Finally, uniform random-walk MCMC sampling of the phase space can be carried out in a way similar to that in asteroid orbital inversion (see Muinonen et al., 2015).

The Lommel–Seeliger reflection coefficient omits the shadowing effects due to the random rough, particulate geometry of the regolith. These effects are accounted for by Wilkman et al. (2015), Muinonen et al. (2011), and Parviainen and Muinonen (2007, 2009). They introduce fractional-Brownian-motion surface roughness on a close-packed plane-parallel medium of finite, spherical particles with a size distribution. In the present work, the particulate–medium reflection coefficient by Wilkman et al. is utilized in asteroid lightcurve-inversion studies via a direct simulation of sparse photometry mimicking that from the Gaia mission.

The LS ellipsoid allows us to study the effect of changing viewing geometry on the photometric phase curves of non-spherical asteroids. We model the LS-ellipsoid phase function with the help of the H, G_1, G_2 phase function developed for asteroid phase-curve studies (Muinonen et al., 2010) and adopted by the International Astronomical Union in 2012 as the new phase function in the official asteroid magnitude system.

The contents of the present work are as follows. In Section 2, we describe the Lommel–Seeliger and particulate–medium reflection coefficients for close-packed media. We give the analytical disk-integrated brightness for the triaxial ellipsoid with a Lommel–Seeliger reflection coefficient. In Section 3, we describe the MCMC methods and, in particular, the concept of the so-called virtual least-squares solutions in defining the MCMC proposal probability densities. Section 4 concentrates on the numerical methods, including a summary for the computation of the particulate–medium reflection coefficient and the robust derivation of least-squares solutions using the downhill simplex method. In Section 5, we apply the inverse methods to both dense and sparse lightcurves, especially, to ground-based observations of main-belt asteroids (e.g., Wang et al., 2015a) and simulations for the Gaia mission (Santana-Ros et al., 2015).

2. Photometry

2.1. Diffuse reflection coefficient

The reflection coefficient R of a surface element relates the incident flux density πF_0 and the emergent intensity I as

$$I(\mu, \phi; \mu_0, \phi_0) = \mu_0 R(\mu, \phi; \mu_0, \phi_0) F_0,$$
$$\mu_0 = \cos\iota, \quad \mu = \cos\epsilon, \tag{1}$$

where ι and ϵ are the angles of incidence and emergence as measured from the outward normal vector of the surface element, and ϕ_0 and ϕ are the corresponding azimuthal angles. It is customary to measure ϕ so that the backscattering direction (or light-source direction) is with $\phi = 0^\circ$. Thus, with the typical assumption of a geometrically isotropic surface, specifying ϕ_0 is unnecessary. The reflection coefficient obeys the reciprocity relation

$$R(\mu, \mu_0, \phi) = R(\mu_0, \mu, 2\pi - \phi). \tag{2}$$

The Lommel–Seeliger reflection coefficient (subscript LS) is (e.g., Lumme and Bowell, 1981)

$$R_{\rm LS}(\mu, \mu_0, \phi) = \frac{1}{4}\tilde{\omega} P(\alpha) \frac{1}{\mu + \mu_0}, \tag{3}$$

where $\tilde{\omega}$ is the single-scattering albedo, P is the single-scattering phase function, and α is the phase angle, the angle between the Sun (light source) and the observer as seen from the object. The Lommel–Seeliger reflection coefficient, the first-order multiple-scattering approximation from the radiative-transfer theory (e.g., Chandrasekhar, 1960), is applicable to weakly scattering media: the intensity terms $[\tilde{\omega}^k]$, $k \geq 2$ are assumed negligible. $\tilde{\omega}$ is the fraction of the incident flux scattered by the single scatterer ($0 \leq \tilde{\omega} \leq 1$). The present single scatterers can be single particles or volume elements within the medium. In scalar radiative transfer omitting polarization effects, the scattering phase function P provides the angular distribution of scattered light in an individual