Planetary and Space Science 101 (2014) 12–26

Contents lists available at ScienceDirect

Planetary and Space Science

journal homepage: www.elsevier.com/locate/pss

On the universal stellar law for extrasolar systems

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ARTICLE INFO

Article history:
Received 20 August 2013
Received in revised form
28 March 2014
Accepted 1 May 2014
Available online 28 May 2014

Keywords:
Prestellar nebula
Statistical theory
Distribution functions
Equation of state of ideal stellar substance
Universal stellar law
Multi-planet extrasolar systems

ABSTRACT

In this work, we consider a statistical theory of gravitating spheroidal bodies to derive and develop an universal stellar law for extrasolar systems. Previously, it has been proposed the statistical theory for a cosmogonic body forming (so-called spheroidal body). The proposed theory starts from the conception for forming a spheroidal body inside a gas-dust protoplanetary nebula; it permits us to derive the form of distribution functions, mass density, gravitational potentials and strengths both for immovable and rotating spheroidal bodies as well as to find the distribution function of specific angular momentum. If we start from the conception for forming a spheroidal body as a protostar (in particular, proto-Sun) inside a prestellar (presolar) nebula then the derived distribution functions of particle as well as the mass density of an immovable spheroidal body characterize the first stage of evolution: from a prestellar molecular cloud (the presolar nebula) to a forming core or a protostar (the proto-Sun) together with its shell as a stellar nebula (the solar nebula). This paper derives the equation of state of an ideal stellar substance based on conception of gravitating spheroidal body. Using this equation we obtain the universal stellar law (USL) for the planetary systems connecting temperature, size and mass of each of stars. This work also considers the solar corona in the connection with USL. Then it is accounting under calculation of the ratio of temperature of the solar corona to effective temperature of the Sun' surface and modification of USL. To test justice of the modified USL for different types of stars, temperature of the stellar corona is estimated. The prediction of parameters of stars is carrying out by means of the modified USL as well as the known Hertzsprung–Russell's dependence is derived from USL directly. This paper also shows that knowledge of some characteristics for multi-planet extrasolar systems refines own parameters of stars. In this connection, comparison with estimations of temperatures using of the regression dependences for multi-planet extrasolar systems testifies the obtained results entirely.

1. Introduction

In 1911–24 the astronomers Russell, Hertzsprung and Eddington established that for stars of the Main sequence there is a dependence of luminosity of a star on temperature of its stellar surface (the diagram of Hertzsprung–Russell), and also there is a connection between luminosity L and mass M of star (the diagram of mass–luminosity) (Eddington, 1916; Jeans, 1929). According to this diagram for stars of the Main sequence, the mass–luminosity dependence looks like $L \propto M^s$, where $s=2.6$ for stars of small masses $(-1.1 < \lg M/M_{Sun} < -0.2)$, $s=4.2$ for stars of medium masses $(-0.2 < \lg M/M_{Sun} < 0.4)$ and $s=3.3$ for stars of big masses $(0.6 < \lg M/M_{Sun} < 1.7)$, M_{Sun} is the mass of the Sun. In the monograph (Krot, 2012a) p.416, the different versions of invariant relations between temperature T, concentration n and parameter of gravitational compression α of Sun-like stars have been derived on the basis of the theory of rotating and gravitating spheroidal bodies. Recently Pintr et al. (2013) have found heuristic regression dependences, i.e. they have studied the regression dependence of the distance of planets a_n from the central stars on the parameter of specific angular momentum $a_n v_n$ (a_n is a planetary distance and v_n is a planetary velocity) and then they have applied the regression analysis to other physical parameters of stars, namely, $a_n T_{eff}$, $a_n L$, and $a_n J$, where T_{eff} is an effective temperature of stellar surface, L is a luminosity of a star, J is a stellar irradiance for the multi-planet extrasolar systems. Thereupon there is a question*: whether there exist like the Kepler's laws an universal law for the planetary systems connecting temperature, size and mass of each of stars?*

According to the statistical theory of gravitating spheroidal bodies (Krot, 2012a, 2012b, 2009) under the usage of laws of celestial mechanics in conformity to cosmogonic bodies (especially, to stars) it is necessary to take into account an extended substance called a *stellar corona*. In this connection the stellar corona can be described by means of model of rotating and gravitating spheroidal body. Moreover, the parameter of gravitational compression α of a

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http://dx.doi.org/10.1016/j.pss.2014.05.002
0032-0633/

spheroidal body (describing the Sun, in particular) has been estimated on the basis of the linear size of its kernel, i.e. *the thickness of a visible part of the solar corona.*

Really, NASA' astronomer Dr. S. Odenwald in his notice “How thick is the solar corona?” wrote: “The corona actually extends throughout the entire solar system as a “wind” of particles, however, the densist parts of the corona is usually seen not more than about 1–2 solar radii from the surface or about 690,000 to 1.5 million kilometers at the equator. Near the poles, it seems to be a bit flatter…” (Odenwald, 1997). In the fact, a recession of plots of dependences of relative brightness of components of spectrum of the solar corona occurs on distance of 3–3.5 radii from the center, i.e. on 2–2.5 radii from the edge of the solar disk.

Thus, accepting thickness of a visible part of the solar corona equal to $\Delta = 2R$ (here R is radius of the solar disk) we find that $r_* = R + \Delta = 3R$, where $r_* = 1/\sqrt{\alpha}$. In other words, the parameter of gravitational compression $\alpha = 1/r_*^2$ of a spheroidal body in case of the Sun with its corona (for which the equatorial radius of disk $R = 6.955 \times 10^8$ m) can be estimated by the value (Krot, 2012a, 2012b):

$$\alpha = \frac{1}{(3R)^2} \approx 2.29701177718 \times 10^{-19} (\mathrm{m}^{-2}). \tag{1}$$

So, the procedure of a finding α is based on the known 3σ-rule in the statistical theory, where $\sigma = 1/\sqrt{\alpha}$ is a root-mean-square deviation of a random variable.

Really, the solar corona accounting under calculation of perturbed orbit of the planet of Mercury allows to find the estimation of a displacement of perihelion of Mercury'orbit for the one period within the framework of the statistical theory of gravitating spheroidal bodies. As it is known, on a way of specification of the law of Newton using the general relativity theory the Mercury problem solving was found (Einstein, 1921). Nevertheless, from a common position of the statistical theory of gravitating spheroidal bodies the points of view as Leverrier (about existence of an unknown matter) and Einstein (about insufficiency of the theory of Newton) practically differ nothing. Really, there exist plasma as well as gas-dust substance around of kernel of cosmogonic body (in particular, the solar corona in case of the Sun), i.e. the account of circumstance that forming cosmogonic bodies have not precise outlines and are represented by means of spheroidal forms demands some specification of the Newton' law in connection with a gravitating spheroidal body.

Using the Binet' formula the equation of disturbed orbit of a planet (the Mercury) in a vicinity of a kernel of a rotating and gravitating spheroidal body has been derived. The obtained relation expresses the equation of the so-called “disturbed” ellipse in polar coordinates with the origin of coordinates in focus, i.e. the planet Mercury is moving on a *precessing elliptic orbit* in view of the fact that there is a modulating multiplier of a phase (or azimuth angle). So, within the framework of the statistical theory of gravitating spheroidal bodies the required angular moving of Newtonian ellipse during one turn of Mercury on the disturbed orbit (or displacement of perihelion of its orbit for the period) has been estimated (Krot, 2012a, 2012b):

$$\delta\varepsilon = \frac{2\pi(3+e)\varepsilon_0^2}{\alpha a^2 (1-e^2)^2}, \tag{2}$$

where through a and e a major semi-axis and an eccentricity of Mercury's orbit are designated respectively, α is a parameter of gravitational compression and ε_0 is a geometrical eccentricity of kernel of a rotating and gravitating spheroidal body (the Sun). Thus, according to the proposed formula (2) the turn of perihelion of Mercury' orbit is equal to 43.93" in century that well is consistent with conclusions of the general relativity theory of Einstein (whose analogous estimation is equal to 43.03") and astronomical observation data (43.11 ± 0.45") (Krot, 2012a, 2012b).

This work also considers the solar corona in the connection with so-called universal stellar law (USL) introduced in the Section 3. Then it is accounting under calculation of the ratio of temperature of the solar corona to effective temperature of the Sun' surface and modification of USL in the next Section 4. To test justice of USL for different types of stars, temperature of the stellar corona is estimated in the Section 5. The Section 6 shows that knowledge of some characteristics for *multi-planet extrasolar systems* permits us to refine own parameters of stars. Really, the numerous papers are devoted to investigations of exoplanetary systems in the last time (for example, (Nottale, 1996; Butler et al., 1999; Agnese and Festa, 1999; Laskar, 2000; Marcy et al., 2001, 2002; Kenyon, 2002; Mayor et al., 2003; De Oliveira Neto et al., 2004, 2005, 2006; Santos et al., 2004; Lovis et al., 2006; Udry et al., 2006; Pepe et al., 2007; Wittenmyer et al., 2007; Döllinger et al., 2007; Sato et al., 2008; Poveda and Lara, 2008; Pintr et al., 2008; Léger et al., 2009; Queloz et al., 2009; Charbonneau et al., 2009; Mayor et al., 2009a, 2009b; Bouchy et al., 2009; Borucki and the Kepler Team, 2010; Borucki et al., 2010; Borucki et al., 2011a, 2011b; Lovis et al., 2011; Kunitomo et al., 2011; Flores-Gutiérrez and Garcia-Guerra, 2011; Fressin et al., 2011; Schneider, 2013)). In this connection, comparison with estimations of temperatures using of the mentioned above regression dependences for multi-planet extrasolar systems testifies the obtained results entirely.

2. The strength, potential and potential energy of the gravitational field of spheroidal body formed by a collection of particles

According to the statistical theory of gravitating spheroidal bodies (Krot, 2012a, 2012b, 2009) the probability density function of a particle having distance r being confined between r and $r + dr$ from the center of a spheroidal body can be expressed by the following formula:

$$f(r) = 4\pi \left(\frac{\alpha}{2\pi}\right)^{3/2} r^2 e^{-\alpha r^2/2}, \tag{3}$$

so that a mass density function of a spheroidal body is equal to

$$\rho(r) = \rho_0 e^{-\alpha r^2/2}, \tag{4}$$

where $\rho_0 = M(\alpha/2\pi)^{3/2}$ is a density in the center of a spheroidal body, M is a mass of a spheroidal body.

Let us calculate the characteristics of the gravitational field produced by a collection of isolated particles in the form of a spheroidal body. We shall use the gravitational field equation in nonrelativistic mechanics written down in the form of the Poisson equation (Landau and Lifschitz, 1951):

$$\Delta\varphi_g = 4\pi\gamma\rho, \tag{5}$$

where Δ is the Laplacian operator, $\gamma = 6.673 \times 10^{-11}$ Nm^2/kg^2 is the Newtonian constant of gravitation, φ_g is a gravitational field potential, ρ is a body mass density. We shall seek a spherically symmetric solution φ_g depending on r only, therefore (5) becomes:

$$\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2 \frac{d\varphi_g(r)}{dr}\right)\right] = 4\pi\gamma\rho_0 e^{-\alpha r^2/2}. \tag{6}$$

Since φ_g is a function of r alone, one obtains:

$$\frac{d\varphi_g(r)}{dr} = 4\pi\gamma\rho_0 \frac{\int_0^r x^2 e^{-\alpha r^2/2}\, dx}{r^2}. \tag{7}$$

On the other hand, the derivative $(d\varphi_g(r))/(dr)$ determines the gradient value $\vec{a}(r) = -grad\varphi_g(r)$, the gradient being also termed the strength of the gravitational field (Landau and Lifschitz, 1951).