Planetary and Space Science 82–83 (2013) 84–98

Contents lists available at SciVerse ScienceDirect

Planetary and Space Science

journal homepage: www.elsevier.com/locate/pss

Iterative alternating sequential (IAS) method for radio tomography of asteroids in 3D

S. Pursiainen [a,b,*], M. Kaasalainen [a]

[a] Department of Mathematics, Tampere University of Technology, Tampere, Finland
[b] Department of Mathematics and Systems Analysis, Aalto University, Helsinki, Finland

ARTICLE INFO

Article history:
Received 21 September 2012
Received in revised form
17 March 2013
Accepted 5 April 2013
Available online 12 April 2013

Keywords:
Asteroids
Radio tomography
Hierarchical Bayesian inversion
IAS algorithm

ABSTRACT

We present a feasibility study of the radio tomography of asteroids. We consider the simplest and most robust type of a radio experiment and physical model, related to the CONSERT (Comet Nucleus Sounding Experiment by Radiowave Transmission) setup, where an orbiter measures the propagation time and amplitude of a radio frequency signal between the orbiter and a transponder placed on an asteroid's surface. Contrary to CONSERT, we consider the simultaneous use of multiple transponders. We study two main questions: (i) what is the basic information content (reconstruction potential) of the data and the minimum number of transponders for recovering most of it and (ii) how to formulate Bayesian methods for an efficient 3D reconstruction. Our approach was to reconstruct the perturbations of a non-constant refractive index inside the asteroid based on simulated signal travel time measurements. We formulate this ill-posed inverse problem by an approximative linear forward (data prediction) model through optical path length and Snell's law, resulting in a formula closely related to the cone-beam and Radon transforms. The linear forward model was applied to three-dimensional asteroid geometries involving an isotropic and piecewise constant refractive index distribution composed of the unknown perturbation and a background given *a priori*. The inverse approach was based on a hierarchical Bayesian model. The reconstructions were produced via the iterative alternating sequential (IAS) maximum a posteriori (MAP) estimation algorithm. We explored the various aspects of the problem by considering the recovery of empty cavities inside an asteroid. Two different transponder setups, a spherical and a realistic computation geometry, as well as various cavity distributions were tested. The results suggest that (i) the information content of the travel time data is robust and allows a unique reconstruction with suitable methods; (ii) finding a reasonable reconstruction requires the use of more than three transponders; (iii) reconstructions with the hierarchical prior model can be superior to those corresponding to Tikhonov regularized solution of the inverse problem; and (iv) producing an appropriate reconstruction necessitates finding a balance regarding the maximal number of reflections taken into account in the forward simulation, in order not to end up with a too sparse or noisy set of data.

1. Introduction

The most readily available methods for studying the interiors of asteroids are radio tomography and seismic experiments. The former is, in principle, best suited for the long-term probing of a sufficiently small target. Low-frequency radio waves can penetrate an asteroid with a diameter of less than a kilometer such that a detectable signal remains. Depending on the experiment, the signals received can be compared with the original ones to reconstruct certain properties and features of the interior. Here, we consider an experimental approach akin to the one used in the ROSETTA (comet rendezvous) mission, but this paper is essentially a mathematical pilot or feasibility study rather than an actual experiment prescription. We want to study the hitherto unexplored basic properties of the inverse problem, especially the information content of the signal from the interior in a simple tomographic radar experiment. In contrast to, e.g., the case of reconstructing the surface from lightcurves and complementary data (Kaasalainen et al., 1992; Kaasalainen and Lamberg, 2006; Kaasalainen, 2011), it is difficult to derive analytical uniqueness and stability theorems for this inverse problem. Thus we have to resort to numerical simulations. Analogously to the lightcurve inversion problem, we considered the simplest type of experiment, data and physical model to find out just how robust the information obtained in radio tomography is.

In addition to inverse problems, experiment design is an important concept. Since we anticipate future space missions, we

* Corresponding author at: Department of Mathematics, Tampere University of Technology, PO Box 553, 33101 Tampere, Finland. Tel.: +358 40 750 5677; fax: +358 9 470 23016.
E-mail addresses: Sampsa.Pursiainen@aalto.fi (S. Pursiainen), mikko.kaasalainen@tut.fi (M. Kaasalainen).

0032-0633/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.pss.2013.04.001

are at liberty to consider various experimental setups. Likewise, the interior of the asteroid does not have to be particularly realistic in simulations: we essentially want to see how well a given internal structure can be reconstructed with different inverse problem setups and solution methods. Considering that a typical space mission costs hundreds of millions of euros, a slight increase in the payload might be acceptable if it would make the information obtained much more valuable.

The overall scenario was chosen to be similar to the CONSERT (Comet Nucleus Sounding Experiment by Radiowave Transmission) experiment (Kofman et al., 1998, 2004, 2007; Herique et al., 1999, 2011a; Barriot et al., 1999) in which an orbiter transmitting and receiving 90 MHz radio frequency signal measures the propagation time and amplitude of the main and secondary paths of a signal emanating from a single transponder located on outer surface of a comet (Nielsen et al., 2001). Regarding tomography of asteroids, this approach to gathering data has been proposed to be used in the ASSERT (ASteroid Sounding Experiment by Radiowave Transmission) experiment (Herique et al., 2011a, 2011b, 2010). In contrast to CONSERT and ASSERT, however, the present study relied on simultaneous use of multiple transponders. The goal was to reconstruct perturbations of a three-dimensional refractive index distribution inside the asteroid based on simulated travel time measurements. This task constituted an ill-posed inverse problem, meaning that the solution was not unique and that finding an appropriate reconstruction necessitated strong *a priori* knowledge (Benna et al., 2002) as well as so-called inversion methods, i.e. computational techniques specifically designed for inverse problems (Kaipio and Somersalo, 2004). The inversion strategy applied was based on the iterative alternating sequential (IAS) algorithm, a fast and easy implementable computational scheme that is well-suited for finding localized reconstructions limited to a small volume (Calvetti et al., 2009; Calvetti and Somersalo, 2008, 2007b).

Central in solving the inverse problem is a forward (data prediction) model, which should take into account, among other things, that a radio signal can be refracted and reflected several times when traveling inside the asteroid. The dependence of the travel time data on the refractive index is, in general, highly nonlinear, since the latter can affect not only the travel time (optical path length) of the signal, but also its direction as follows from Snell's law (Hecht, 2002). If, however, only a small-sized perturbation of the refractive index is to be sought, the forward model can be linearized by assuming that the perturbation does not affect the direction. In this study, the resulting linear operator, closely related to the cone-beam and Radon transform (Barriot et al., 1999), was applied in 3D to a set of ray paths emanating from the transponders and predicted by Snell's law. The inverse approach adopted was a hierarchical Bayesian one in which a posterior probability density of the refractive index was formed as a product of a likelihood of the data, a prior density conditional on one or more hyperparameters, as well as a hyperprior, i.e. the prior of the hyperparameters (O'Hagan and Forster, 2004). Gamma and inverse gamma hyperpriors were utilized to produce well-localized results (Calvetti et al., 2009; Calvetti and Somersalo, 2007b, 2008). Based on the posterior, the eventual reconstruction was found as a *maximum a posteriori* (MAP) estimate via the IAS algorithm.

To test the performance of the approach, we studied the recovery of internal cavities (inclusions) that were assumed to be spatially well-localized but to differ considerably in terms of the refractive index, e.g., due to hollowness, as compared to the background. This scenario was chosen to represent the discontinuous aspects of the problem rather than to simulate a specific geological condition, but the existence of significant internal cavities has been suggested based on the results concerning binary near earth asteroids (NEAs; see, e.g., Belton, 2004, p. 143; Scheirich and Pravec, 2009 and references therein). These show that the bulk density of an asteroid can be relatively low as compared to the density of minerals it is composed of Belton (2004). The recovery of localized but intense perturbations is challenging with respect to both the linear forward model and the inverse problem, which, with a slightly different formulation, have previously been investigated in the case of faint perturbations, e.g., in Landmann et al. (2010) and Barriot et al. (1999). In addition, the combination of the forward and inverse methods utilized is a novel one in the current context of radio tomography.

Producing an accurate reconstruction based on travel time data from a single transponder is known to be extremely difficult: for example, a forward operator for the travel time can suffer from a non-trivial null space (Landmann et al., 2010; Barriot et al., 1999; Golub and van Loan, 1989). To improve the inversion results, the forward simulation can be complemented with signal amplitude (Arnold et al., 2011; Landmann et al., 2010; Carley and Heggy, 2008; Benna et al., 2004; Piot, 2002; Barriot et al., 1999) in parallel with CONSERT (Kofman et al., 1998, 2007). As a feasibility study, this work concentrated merely on the travel time, which is more directly dependent on the refractive index and known to be less sensitive with respect to modeling errors than the amplitude (Landmann et al., 2010; Barriot et al., 1999). To obtain appropriate results, the forward model incorporated data from multiple transponders, which is a potential scenario regarding future missions.

In the numerical experiments, the recovery of cavities was carried out by using different numbers of transponders, a spherical and a realistic (randomized) computation geometry, as well as various cavity distributions. In particular, the aim was to address the following questions: (i) What would be an appropriate number of transponders to be used simultaneously; (ii) How does the present reconstruction strategy compare with Tikhonov regularization (Kaipio and Somersalo, 2004) applied previously, e.g., in Barriot et al. (1999); and, (iii) What would be a reasonable number of full reflections allowed for the signal in the simulation of the forward model.

In this paper, Section 2 describes the forward model and its simulation, a hierarchical Bayesian inverse approach, and the IAS algorithm. It also motivates the simultaneous use of multiple transponders. Section 3 gives details of the experimental setups and reports the results obtained. In Section 4 we discuss the results and consider some future directions, and we sum up in Section 5.

2. Materials and methods

2.1. Forward model

The present study assumed that the refractive index n of the target asteroid Ω is a piecewise constant function of the form $n = n_p + n_b$ in which n_p is a localized perturbation to be sought, and n_b is a background distribution known *a priori*. Travel time τ of a signal is given by the optical path length (Born and Wolf, 1999), that is, the refractive index integrated over the signal (ray) path $\mathcal{C}$ as given by

$$\tau = \int_{\mathcal{C}\cap\Omega} n_p \, ds + \int_{\mathcal{C}} n_b \, ds + \text{noise}, \tag{1}$$

here the path $\mathcal{C}$, which formally follows from the eikonal equation $(\nabla\tau)^2 = n^2$ (Born and Wolf, 1999), was traced by applying Snell's law (Born and Wolf, 1999) on the discontinuity surfaces of the refractive index. To avoid an *inverse crime*, i.e. an overly optimistic fit (Kaipio and Somersalo, 2004), ray tracing was done based on two different distributions, n and n_b, in generation of the source