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Dispersive MHD turbulence in one dimension

D. Laveder, L. Marradi, T. Passot, P.L. Sulem *

Université de Nice Sophia Antipolis, CNRS, Observatoire de la Côte d'Azur, BP 4229, 06304 Nice Cedex 4, France

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ABSTRACT

Numerical simulations of dispersive turbulence in magnetized plasmas based on the Hall-MHD description are presented, assuming spatial variations along a unique direction making a prescribed angle with the ambient magnetic field. Main observations concern the energy transfers among the different scales and the various types of MHD waves, together with the conditions for the establishment of pressure-balanced structures. For parallel propagation, Alfvén-wave transfer to small scales is strongly inhibited and rather feeds magnetosonic modes, unless the effect of dispersion is strong enough at the energy injection scale. In oblique directions, the dominantly compressible character of the turbulence is pointed out with, for quasi-transverse propagation, the presence of conspicuous kinetic Alfvén waves. Preliminary simulations of a Landau fluid model incorporating relevant linear kinetic effects reveal the development of a significant plasma temperature anisotropy leading to recurrent instabilities.

1. Introduction

Turbulence in magnetized plasmas remains a main issue in the understanding of the dynamics of media such as the solar corona, the interstellar medium, the solar wind or the planet magnetosheaths. In the solar wind for example the turbulent cascade extends much beyond the ion Larmor radius. One of the questions concerns the spectrum of the magnetic fluctuations that displays a power-law behavior on a broad range of wavenumbers, with a conspicuous change of slope near the inverse ion gyroradius (Leamon et al., 1998; Golstein and Roberts, 1999; Alexandrova et al., 2006; Sahraoui et al., 2009). This effect is often associated with the influence of wave dispersion, induced by the Hall current (Ghosh et al., 1996; Galtier, 2006; Alexandrova et al., 2007; Galtier and Buchlin, 2007; Servidio et al., 2007; Shaikh and Shukla, 2009), but could also result from a superposition of cascades of kinetic Alfvén waves and ion entropy fluctuations, as suggested by studies based on the gyrokinetic formalism (Howes et al., 2008b, 2008a; Schekochihin et al., 2009).

At scales large compared with the ion inertial length or the ion Larmor radius, the usual MHD description provides a satisfactory description of regimes where, due to the presence of a strong ambient field, a dominant effect is the anisotropic energy transfer to Fourier modes with large transverse wavenumbers (see e.g. Ghosh and Golstein, 1997; Oughton and Matthaeus, 2005 and references therein). This suggests that the dynamics of transverse small scales may be amenable to a reduced MHD description ((Zank and Matthaeus, 1992) and references therein), possibly including Hall current (Gómez et al., 2008) or, when retaining scales significantly smaller than the ion Larmor radius, to a gyrokinetic approach (Howes et al., 2006; Schekochihin et al., 2009). The latter that appears to be very efficient in describing strongly magnetized near-equilibrium fusion plasmas is still under discussion concerning its applicability to space and astrophysical plasmas (Matthaeus et al., 2008). In the solar wind for example magnetic fluctuations may be comparable to the ambient field. Furthermore, longitudinal transfer could a priori be non-negligible in a compressible regime, at scales where Hall current and kinetic effects play a significant role. A weak turbulence theory performed on the Vlasov–Maxwell system was recently developed (Yoon and Fang, 2008), showing the existence of a parallel cascade of low-frequency Alfvén waves through a three-wave decay process mediated by ion-sound turbulence, in a regime where wave-particle interactions are neglected. Addressing this issue by direct numerical simulations of the Vlasov–Maxwell equations being still difficult on the present-day computers, the question arises whether a similar cascade can be observed within a fluid model that retains important ingredients of the above theory, such as compressibility and dispersion. As a first step, we address the problem within the simplest description provided by Hall-MHD (HMHD) with Ohmic and viscous dissipations, together with a large-scale external driving acting on the transverse components of the velocity or magnetic field. We specifically concentrate on a one-dimensional setting where the variations of the fields are

* Corresponding author. Tel.: +33 4 92 00 30 40; fax: +33 4 92 00 31 21.
E-mail addresses: laveder@oca.eu (D. Laveder), marradi@oca.eu (L. Marradi), passot@oca.eu (T. Passot), sulem@oca.eu (P.L. Sulem).

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restricted to a direction making a prescribed angle with the ambient magnetic field, a framework that already reveals a manifold of complex dynamical processes that deserve detailed investigations before including additional physical and multi-dimensional effects. In the case of quasi-transverse propagation, we also present simulations of a model that extends the HMHD by retaining pressure anisotropy, Landau damping and finite Larmor radius effects up to transverse scales significantly smaller than the ion Larmor radius. This approach developed in Passot and Sulem (2007) extends the so-called Landau fluid model initiated in Snyder et al. (1997) for the MHD scales where Landau damping is the only relevant kinetic effect.

The paper is organized as follows. Section 2 briefly reviews the Hall-MHD description and its one-dimensional reduction. Section 3 concentrates on the case where the dynamics takes place in the direction of the ambient field. The case of oblique propagation is addressed in Section 4. Landau fluid simulations retaining small-scale kinetic effects are reported in Section 5. Our conclusions are summarized in Section 6.

2. The Hall-MHD description

HMHD can be viewed as a bi-fluid description of a plasma, where electron inertia is neglected. The presence of the Hall term in the generalized Ohm's law allows a decoupling of the ion fluid from the electron one in which the magnetic field lines are frozen. The validity conditions of HMHD are discussed in Howes (2009) where comparisons with kinetic theory are presented. Choosing as units the Alfvén speed, the amplitude of the ambient magnetic field, the equilibrium density and the ion inertial length l_i (defined as the ratio of the Alfvén speed to the ion gyrofrequency), the HMHD equations (for the ion fluid) read

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho(\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v}) = -\frac{\beta}{\gamma}\nabla \rho^{\gamma} + (\nabla \times \mathbf{b}) \times \mathbf{b} \tag{2}$$

$$\partial_t \mathbf{b} - \nabla \times (\mathbf{v} \times \mathbf{b}) = -\nabla \times \left(\frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b}\right) \tag{3}$$

$$\nabla \cdot \mathbf{b} = 0, \tag{4}$$

where the total β parameter is the square ratio of the sound to Alfvén velocities, and a polytropic equation of state $p \propto \rho^{\gamma}$ is assumed for both ions and electrons.

When the spatial variation is restricted to a dependency on the x coordinate along a direction making an angle θ with the ambient magnetic field $\mathbf{B}_0 = (\cos\theta, \sin\theta, 0)$, one gets

$$\partial_t \rho + \partial_x(\rho v_x) = 0 \tag{5}$$

$$\partial_t v_x + v_x \partial_x v_x = -\frac{1}{\rho}\partial_x\left(\frac{\beta}{\gamma}\rho^{\gamma} + \frac{1}{2}(b_y^2 + b_z^2)\right) + \frac{\mu_x}{\rho}\partial_{xx} v_x \tag{6}$$

$$\partial_t v_{[y,z]} + v_x \partial_x v_{[y,z]} = \frac{\cos\theta}{\rho}\partial_x b_{[y,z]} + \frac{\mu_{[y,z]}}{\rho}\partial_{xx} v_{[y,z]} + f^v_{[y,z]} \tag{7}$$

$$\partial_t b_{[y,z]} - b_x \partial_x v_{[y,z]} + \partial_x(v_x b_{[y,z]}) = \pm\cos\theta\,\partial_x\left(\frac{1}{\rho}\partial_x b_{[z,y]}\right) + \kappa_{[y,z]}\partial_{xx} b_{[y,z]} + f^b_{[y,z]}, \tag{8}$$

where driving and dissipation have been supplemented in both the velocity and magnetic field equations. Here, the subscript $[y,z]$ refers to the vector component along the y or the z direction, or to the value of the viscosity acting on the corresponding velocity component. The $\pm$ sign in front of the Hall term depends on the considered component of the magnetic field. No (artificial) hyperviscosity and magnetic diffusivity nor spectral filtering are used in the simulations. Instead, anisotropic dissipations are assumed. In the case of parallel propagation, different viscosities and diffusivities are taken in the directions parallel and transverse to the ambient field, by prescribing $\kappa_y = \kappa_z = \mu_y = \mu_z \ll \mu_x$. For oblique propagation, we assume smaller coefficients in the direction perpendicular to the plane defined by the magnetic field and the direction of propagation, in the form $\kappa_z = \mu_z \ll \mu_x = \mu_y = \kappa_y$.

The driving is assumed to act either on the velocity (kinetic driving) or the magnetic field (magnetic driving) components. For parallel propagation, we prescribe $f^v_y = f^v_z$ or $f^b_y = f^b_z$ while, for oblique propagation, the driving reduces to f^v_z or to f^b_z. Such a driving is supposed to minimize the sonic components, as it is acting on field components perpendicular to the ambient field in parallel propagation and to the plane defined by the ambient field and the propagation direction when the latter is oblique. The values of the diffusivity and viscosities in the various directions are chosen as the minimal values (depending on the spatial resolution and of the physical parameters of the runs) needed to accurately resolve all the retained scales.

In all the simulations, we take $\gamma = 5/3$ and $\beta = 2$. Each component of the kinetic or magnetic driving (generically denoted f) is a white noise in time defined by its Fourier transform $\hat{f}_k = C\xi\sqrt{F_k/\Delta t}$ where ξ is a Gaussian random variable with zero mean and unit variance, chosen independently at each time step. This ensures a constant mean flux of energy injection that can be chosen at will, as in the usual phenomenology of the turbulent cascades. Furthermore, such a driving process avoids an artificial enhancement of a specific type of waves and enables the emergence of the dominant modes as the result of the nonlinear dynamics. The spectral distribution $F_k = k^4\exp(-(2k^2/k_f^2))$ is peaked about a wavenumber k_f.

The HMHD system is integrated in a periodic domain using a Fourier pseudo-spectral method where most of the aliasing is removed by spectral truncation of the computed nonlinear terms at 2/3 of the maximal wavenumber. The spatial resolutions given in the following sections are the effective ones, after aliasing has been suppressed. In all the simulations, the temporal scheme is a third-order low-storage Runge–Kutta (Williamson, 1980). Resolving all the temporal scales present in the system, this scheme accurately preserves the dispersion relation of all the linear modes retained in the simulation, in contrast with implicit or semi-implicit schemes (Laveder et al., 2009).

For convenience, we collected in Table 1, the main parameters characterizing the simulations discussed in the forthcoming sections.

As seen in the following, in spite of the turbulent regime achieved in the HMHD simulations discussed in this paper, signatures of the linear waves are often present. It is thus useful to briefly review the linear theory of eigenmodes for the HMHD equations in the absence of dissipation and driving. By linearizing Eqs. (5)–(8) about the equilibrium state associated to $\rho = 1$, $b_x = \cos\theta$, $b_y = \sin\theta$, one derives that the (real) eigenfrequencies

Table 1
Simulation parameters of HMHD simulations.

Run	Domain size	Propagation angle (°)	driving
A	$L = 16\pi$	$\theta = 0$	Kinetic, C=0.1, $k_f\, l_i$=1/2
B	$L = 16\pi$	$\theta = 0$	Magnetic, C=0.1, $k_f\, l_i$=1/2
C	$L = 4\pi$	$\theta = 0$	Kinetic, C=6.25 $\times 10^{-3}$, $k_f\, l_i$=2
D	$L = 4\pi$	$\theta = 0$	Magnetic, C=6.25 $\times 10^{-3}$, $k_f\, l_i$=2
E	$L = 16\pi$	$\theta = 45$	Kinetic, C=0.1, $k_f\, l_i$=1/2
F	$L = 4\pi$	$\theta = 45$	Kinetic, C=6.25 $\times 10^{-3}$, $k_f\, l_i$=2
G	$L = 4\pi$	$\theta = 45$	Magnetic, C=6.25 $\times 10^{-3}$, $k_f\, l_i$=2
H	$L = 16\pi$	$\theta = 80$	Kinetic, C=0.1, $k_f\, l_i$=1/2