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# Non-Maxwellian effects in magnetosonic solitons

O.A. Pokhotelov[a,*], M.A. Balikhin[a], O.G. Onishchenko[b], S.N. Walker[a]

[a]Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK
[b]Institute of Physics of the Earth, Moscow, Russia

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**Abstract**

The role of non-Maxwellian effects on magnetosonic (MS) solitons propagating perpendicular to the external magnetic field in high-$\beta$ plasmas is analysed. It is shown that they can exist in the form of either humps or holes in the magnetic field in which the field is either increased or decreased relative to the background magnetic field. The shape of the solitary structure depends upon both the form of the ion velocity distribution function and the wave dispersion. A nonlinear equation describing the propagation of MS solitons in high-$\beta$ plasmas with an arbitrary particle velocity distribution function is derived. It is shown that for Maxwellian and bi-Maxwellian plasmas MS solitons can only exist in the form of the magnetic humps. The same is true for plasmas possessing either a kappa distribution or Kennel–Ashour–Abdalla equilibria. However, plasmas with a ring type ion velocity distribution or a Dory–Guest–Harris distribution with large loss-cone index can support the formation of magnetic holes. The theoretical results obtained are then compared with recent satellite observations.


*Keywords:* Magnetosonic waves; Solitons; Mirror instability; Magnetosheath

## 1. Introduction

It is of common knowledge that space plasma particle distributions can substantially deviate from the standard Maxwellian shape. The simplest non-Maxwellian distribution is the bi-Maxwellian distribution exhibiting different temperatures parallel and perpendicular to the magnetic field. Furthermore, energetic ions and electrons, often modelled by $\kappa$ and/or loss-cone distributions, are also ubiquitously observed in space. Various particle acceleration mechanisms have been proposed to explain such non-Maxwellian effects. These effects can substantially influence the propagation of large amplitude magnetosonic (MS) waves which are frequently observed in the upstream and downstream regions of the terrestrial bow shock (Balikhin et al., 1997; Narita et al., 2006). They are also inherent in the magnetosheath, i.e. the turbulent region between the bow shock and the magnetopause. The magnetosheath contains a large number of MS solitary structures of various shapes. Some of them belong to the class of magnetic holes propagating at large angles to the ambient magnetic field whereas others are disturbances with increased magnetic fields, i.e. the magnetic humps (Lucek et al., 2005). There have been numerous previous analytical and numerical studies of nonlinear structures involving MS waves (e.g., Kaufman and Stenflo, 1975; Yu and Shukla, 1977; Baumgärtel, 2005). Large amplitude MS waves propagating at large angles to the ambient magnetic field have been extensively studied as far back as the end of 1960s by Kennel and Sagdeev (1967). These authors predicted that for high-$\beta$ Maxwellian plasmas MS waves can propagate as rarefaction solitons, i.e. in the form of magnetic holes. This analysis, however, has been revisited by Macmahon (1968) in which it was shown that MS solitons in Maxwellian plasmas in fact correspond to the magnetic humps. A further discussion of this problem has been provided by Mikhailovskii and Smolyakov (1985). The present paper develops a fully kinetic analysis of MS waves in a high-$\beta$ plasma, taking into consideration both

*Corresponding author. Tel.: +44 1142225197; fax: +44 1142225661.
*E-mail address:* o.a.pokhotelov@sheffield.ac.uk (O.A. Pokhotelov).

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non-Maxwellian distributions and finite amplitude effects. This theory can then be used to provide an answer to the question regarding the conditions required for MS waves to propagate in the form of magnetic humps or holes.

The paper is structured as follows: Section 2 outlines the development of a general linear dispersion relation for MS waves in a non-Maxwellian plasma. The equation that governs the nonlinear dynamics of MS solitons in non-Maxwellian plasmas is obtained in Section 3. Finally, the results are discussed in Section 4.

## 2. MS linear dispersion relation

In this section, an expression for the linear dispersion relation for MS waves propagating in the direction perpendicular to the magnetic field is deduced for a high-$\beta$ plasma with an arbitrary particle distribution. A local Cartesian system of coordinates is used with the $z$-axis defined by the external magnetic field direction and the $x$-axis is defined by the direction of the wave vector $\mathbf{k}$. The $y$-axis completes the triad. All perturbed quantities are assumed to vary as $\exp(-\mathrm{i}\omega t + \mathrm{i}kx)$. For further simplicity the analysis is restricted to the most important case in which the ion temperature is much larger than the electron temperature.

Low frequency MS waves propagating perpendicular to the external magnetic field in high-$\beta$, uniform plasmas can be described by the following system of equations (Pokhotelov et al., 1985):

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} - N^2 \end{pmatrix} \begin{pmatrix} \Psi_\mathrm{A} \\ \Psi_\mathrm{M} \end{pmatrix} = 0, \tag{1}$$

where $\varepsilon_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ are the components of the dielectric tensor given by

$$\varepsilon_{22} \simeq \frac{c^2}{v_\mathrm{A}^2}\left(1 + \frac{\omega^2}{\omega_{ci}^2} - \frac{11}{4}k^2\rho_i^2\right) - N^2\left(\beta_\perp - \frac{3}{2}\lambda\beta_\perp k^2\rho_i^2\right), \tag{2}$$

$$\varepsilon_{12} = -\varepsilon_{21} \simeq \mathrm{i}\frac{c^2}{v_\mathrm{A}^2}\frac{\omega}{\omega_{ci}}\left(1 - \frac{3}{2}\frac{\omega_{ci}^2}{\omega^2}k^2\rho_i^2\right), \tag{3}$$

$$\varepsilon_{11} \simeq \frac{c^2}{v_\mathrm{A}^2}\left(1 + \frac{\omega^2}{\omega_{ci}^2} - \frac{3}{4}k^2\rho_i^2\right), \tag{4}$$

where the refractive index $N$ is given by $N = kc/\omega$, $\omega$ is the wave frequency, $c$ the speed of light, $k$ the wave number, $\omega_{ci} = eB/m$ is the ion cyclotron frequency, $e$ is the ionic charge, $m$ is the ion mass, $v_\mathrm{A} = B/(\mu_0\rho)^{1/2}$ is the Alfvén velocity, $\mu_0$ is the permeability of free space, $\rho$ is the plasma mass density, $\rho_i = (p_\perp/\rho)^{1/2}/\omega_{ci}$ is the ion Larmor radius, $\beta_\perp \equiv 2\mu_0 p_\perp/B^2$ is the perpendicular plasma beta, and $p_\perp$ is the plasma pressure. The parameter $\lambda$ is given by

$$\lambda = \frac{n\langle W^2 F\rangle}{2(p_\perp)^2}, \tag{5}$$

where $n$ is the plasma number density, $W = mv_\perp^2/2$ is the perpendicular ion energy and $F$ is the ion velocity distribution function. The angular brackets $\langle\cdots\rangle$ denote averaging over the full velocity space (i.e. $\langle\cdots\rangle = \int \mathrm{d}\mathbf{v}(\cdots)$). The potentials $\Psi_\mathrm{A}$ and $\Psi_\mathrm{M}$ are connected with the perpendicular components of the electric field $E_x$ and $E_y$ and the compressional component of the magnetic field $\delta B_z$ by the relations $\Psi_\mathrm{A} = -\mathrm{i}E_x/\omega$ and $\Psi_\mathrm{M} = -\mathrm{i}E_y/\omega = \mathrm{i}\delta B_z/k$.

By setting the determinant of the system of equations (1) equal to zero, we obtain the dispersion relation that determines the eigenmode spectrum for the MS waves in a magnetized plasma:

$$N^2 = \varepsilon_{22} + \varepsilon_{12}^2\varepsilon_{11}^{-1}. \tag{6}$$

Substituting expressions (2)–(4) into Eq. (6) one finds the MS dispersion relation

$$\omega^2 = k^2 V_\mathrm{A}^2(1 - k^2 d^2), \tag{7}$$

where $V_\mathrm{A} = v_\mathrm{A}(1 + \beta_\perp)^{1/2}$ is the generalized Alfvén velocity that includes the effects of a high-$\beta$ plasma and $d$ is the dispersion length given by

$$d^2 = \frac{\rho_i^2}{4}\left(\frac{6\lambda - \frac{7}{2} + \beta_\perp^{-1}}{1 + \beta_\perp^{-1}}\right). \tag{8}$$

From Eq. (8) it follows that the square of the dispersion length can be either positive or negative depending on the shape of the ion distribution function which is defined by the sign of the numerator. The MS dispersion is positive if the parameter $\lambda$ is relatively small, i.e. $\lambda < \frac{7}{12}$. This is a necessary (but not sufficient) condition for positive dispersion. For larger values of $\lambda$ the square of dispersion length is negative.

At this point a few comments are in order. For a canonical Maxwellian distribution function $\lambda = 1$ and $d^2 > 0$. As a result, the MS dispersion is negative as was claimed by Macmahon (1968) who used a simplified fluid approach. As will be shown in the next section this can result in the appearance of magnetic humps in high-$\beta$ plasmas. The same is true for bi-Maxwellian distributions $F \propto \exp(-v_\parallel^2/v_{T_\parallel}^2 - v_\perp^2/v_{\mathrm{T}_\perp^2})$ for which the parameter $\lambda$ is also unity and $d^2$ is given by

$$d^2 = \frac{\rho_i^2}{4}\left(\frac{1 + \frac{5}{2}\beta_\perp}{1 + \beta_\perp}\right). \tag{9}$$

Thus, even in non-equilibrium plasmas that possess a bi-Maxwellian distribution, the dispersion of MS waves is still negative, i.e. the phase velocity decreases with increasing wave number.

A positive dispersion can appear for a plasma with an ion ring-type distribution $F_i \propto \delta(v_\perp - v_{\perp 0})$, where $v_{\perp 0}$ is the ion "ring velocity". For such distributions $\lambda = \frac{1}{2}$ and the necessary condition for the reversal of the sign of $d^2$ is satisfied, if $\beta_\perp > 2$. Such distribution functions were systematically observed when MS waves were detected in the Earth's magnetosphere by the GEOS 1 and 2 satellites (Perraut et al., 1982). A ring type ion distribution can also be formed during the pick up process when neutral atoms