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Decay of orbits due to the drag of rotating oblate atmosphere

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Abstract

In the present work, the orbit prediction with the KS regular variables was obtained numerically. Perturbations of an artificial satellite due to the non-aspherecity of the Earth and the atmospheric drag are considered. The Earth's zonal harmonic terms are included up to J_6, and a model of drag with an oblate rotating atmosphere was obtained. The variation of the rotational velocity of the atmosphere is taken into account. The theory obtained has been applied to study the orbital decay of the near-Earth Indian Satellite RS-1. We compared our results with those of Sharma and Mani [1985. Study of RS-1 orbital decay with KS differential equations. Indian J. Pure Appl. Math. 16(7), 833–842] and Sehnal and Pospisilova [1991. Lifetime of the Rohini a satellite. Bull. Astron. Inst. Czechosl. 42, 295–299].

Keywords: Space dynamics; Orbits perturbation; Artificial satellite; Drag force and KS equations

1. Introduction

To match with the present operational requirements and observational techniques, it has become necessary to use extremely complex force models for precise orbit computation of artificial Earth satellites. In the near-Earth environment the problem becomes more complicated due to the fact that the satellite is influenced by the non-spherical effects of the Earth's gravitational field as well as the dissipative effects of the Earth's atmosphere. It is very difficult to determine the effects of the atmosphere since the atmospheric density and hence the drag undergo large space and time-dependent fluctuations that may or may not be modeled correctly. One must select a mathematical representation for these forces and must decide on a mathematical solution technique for integrating the resulting differential equations of motion.

The modeling of drag perturbations is still a topic of active research. The options for mathematical solution techniques can be classified as analytical, semi-analytical and numerical methods. Both analytical and semi-analytical methods use an analytical transformation to produce mean, slowly varying differential equations. Stern (1959, 1960) and Cook et al. (1961) had generated analytical solutions for near-Earth low eccentricity orbits. Kampose (1968) carried out an extension of the work reported by Kalil (1968) with the same atmosphere model. King-Hele (1962) and King-Hele and Walker (1987) extended the solutions to high-eccentricity orbits. Engels and Junkins (1981) and Jezewski (1983) also evolved analytical solutions with J_2 for short-term orbit. Analytical solutions with air drag and oblate atmosphere for near-Earth satellite orbits were obtained by Sharma (1991, 1992, 1997, 1998). However, the numerical integration methods provide more accurate ephemeris of a satellite with respect to any type of perturbing forces (Merson, 1975; Graf et al., 1977; Sharma and Mani, 1985). But these methods proved to be costly in terms of computer time, and at times are of questionable validity in high-accuracy regions.

It is well known that the solutions of the classical Newtonian equations of motion are not suitable for long-term integration. Many transformations have emerged in the literature in the recent past to stabilize the equations of motions either to reduce the accumulation of local numerical errors or allowing the use of larger integration

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step sizes, in the transformed space or both (Sharma and Mani, 1985). Examples of such transformations include the use of a new independent variable, time transformation to orbital parameter space that tends to decouple fast and slow variables, and the use of integrals as control terms. One such transformation, known as the KS transformation, is due to Kustaanheimo and Stiefel (1965), who regularized the nonlinear Kepler motion and reduced it to linear differential equations of a harmonic oscillator of constant frequency. Stiefel and Scheifele (1971) further developed the application of the KS transformation to problems of perturbed motion, producing a perturbation equations version.

In the present paper, we have studied the orbital decay of the Indian Satellite RS-1 which was launched from Sriharikota range on 18 July 1980. The mathematical model included the Earth's zonal harmonics up to J_6 and the oblate rotating atmospheric drag has been taken into account. We have integrated the KS perturbation equations of motion. Starting from 20 July 1980 epoch (Sharma and Mani, 1985) the package has been carried out with fourth-order Runge–Kutta method and Mathematica software (Wolfram version5). The results had been compared with those of Sharma and Mani (1985) and Sehnal and Pospisilova (1991).

2. The KS transformation

It is well known that the equation of motion of an artificial satellite under the effect of the Earth's potential and any given perturbing force is defined as

$$\ddot{\bar{x}} + \frac{\mu}{r}\bar{x} = -\frac{\partial V}{\partial \bar{x}} + \bar{P}, \tag{1}$$

where $V(t, x)$ is the perturbing potential depending on time and the position of the satellite, including the asphericity of the Earth and the gravitational effect. $\bar{P}$ is the perturbing force produced by drag, radiation pressure or third body attraction, etc., and $r = |\bar{x}|$.

Now, in order to use the KS transformation, the independent variable is changed from t (the ordinary time) to s (the fictitious time) through the following equation:

$$\mathrm{d}t = r\,\mathrm{d}s, \tag{2}$$

where r is the geocentric distance of the satellite, and the vector $\bar{x}$ is a dependent variable, and is changed to a four-vector $\bar{u}$ composed of the components u_j ($j = 1$–4):

$$\bar{x} = L(u)\bar{u}, \tag{3}$$

where

$$L(u) = \begin{pmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix}. \tag{4}$$

This may be written explicitly as

$$x_1 = u_1^2 - u_2^2 - u_3^2 + u_4^2, \tag{5.1}$$

$$x_2 = 2(u_1u_2 - u_3u_4), \tag{5.2}$$

$$x_3 = 2(u_1u_3 + u_2u_4). \tag{5.3}$$

From these equations it follows that

$$r^2 = x_1^2 + x_2^2 + x_3^2 = (u_1^2 + u_2^2 + u_3^2 + u_4^2)^2. \tag{6}$$

From Eq. (5) we can evaluate $\dot{\bar{x}}$ and $\ddot{\bar{x}}$. Notice that the prime symbol indicates the differentiation with respect to s, and the dot differentiation with respect to t. Then the equation of motion (1), after some reductions and some properties of the vector, is thus transformed into

$$u'' + \frac{h}{2}u = -\frac{1}{4}\frac{\partial}{\partial u}(|u|^2 V) + \frac{|u|^2}{2}L^{\mathrm{T}}\bar{P}, \tag{7}$$

$$h = h_k - V, \quad h_k = \frac{\mu}{r} - \frac{1}{2}\dot{\bar{x}}^2,$$

where h is the total energy and h_k is the Keplerian energy.

In the case of pure Kepler motion (unperturbed case), the equation of motion (7) reduced to

$$u'' + \frac{h}{2}u = 0,$$

and the general solution is given by

$$\bar{u} = \bar{\alpha}\cos ws + \bar{\beta}\sin ws, \tag{8}$$

$$w = \sqrt{\frac{h}{2}} = \frac{E}{2s}, \tag{9}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are victorian constants of integration, w is the frequency, h is the total energy and the angle E is the eccentric anomaly.

Since the motion is perturbed, the equations of motion are now solved by the familiar method of variation of constants. This technique leads to a system of differential equations for the functions $\bar{\alpha}(E)$, $\bar{\beta}(E)$, $w(E)$ and $\tau(E)$.

3. The equations of motion

The KS element equations of motion of a satellite under the effect of additional perturbing force $\bar{P}$ (Stiefel and Scheifele, 1971) are

$$\frac{\mathrm{d}w}{\mathrm{d}E} = -\frac{r}{8w^2}\frac{\partial V}{\partial t} - \frac{1}{2w}(\bar{u}^*, L^{\mathrm{T}}\bar{P}),$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}E} = \frac{1}{8w^3}(\mu - 2rV) - \frac{r}{16w^3}\left(u, \left(\frac{\partial V}{\partial \bar{u}} - 2L^{\mathrm{T}}\bar{P}\right)\right) - \frac{2}{w^2}\frac{\mathrm{d}w}{\mathrm{d}E}\left(\bar{u}, \frac{\mathrm{d}\bar{u}}{\mathrm{d}E}\right),$$

$$\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}E} = \left\{\frac{1}{2w^2}\left[\frac{V}{2}\bar{u} + \frac{r}{4}\left(\frac{\partial V}{\partial \bar{u}} - 2L^{\mathrm{T}}\bar{P}\right)\right] + \frac{2}{w}\frac{\mathrm{d}w}{\mathrm{d}E}\frac{\mathrm{d}\bar{u}}{\mathrm{d}E}\right\}\sin\frac{E}{2},$$