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Behavior of the Rayleigh–Taylor mode in a dusty plasma with rotational and shear flows

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Abstract

The stability of a dusty plasma with sheared rotational flows is investigated. Using the fluid model together with the Bayly nonmodal approach, the inhomogeneous partial differential equations governing short-wavelength perturbations at the center of a rotational flow field or vortex structure are obtained. The effects of flow eccentricity, strength of the flow shear, as well as concentration of dust grains on the stability of the perturbations are investigated numerically. It is found that flow shear can cause secondary Rayleigh–Taylor instability of a rotational flow.

Keywords: Rotational flow; Sheared flow; Rayleigh–Taylor mode; Dust grains; Complex plasma

1. Introduction

Rotational and sheared flows along and across the Earth's magnetic field are common in the ionosphere. Being a possible source of free energy, such plasma flows can significantly affect the dynamics and stability behavior of the plasma (Sekar and Raghavarao, 1987; Bonnell, 1996; Mendillo et al., 2001; Kopnin et al., 2004; Popel et al., 2004a, b). Similarly, sheared rotational flows exist in most toroidal magnetic confinement devices such as the tokamak, and they have been associated with the ubiquitous low-frequency micro-turbulence and can strongly affect the plasma transport properties (Tynan et al., 1992). Recently, there have also been much interest in large-scale rotational vortex-like structures involving sheared flows (Bonnell, 1996; Changelishvili et al., 1997; Vranjes, 1999; Chakrabarti, 2000; Mikhailenko et al., 2002; Chen et al., 2003; Kim et al., 2005; Siefring and Bernhardt, 2005). On the other hand, many low-temperature plasmas in space (Goertz, 1989; Verheest, 2000; Vranjes et al., 2003; Kopnin et al., 2004) and the laboratory (Narihara et al., 1997; Ostrikov et al., 2000a, b; Winter, 2000; Xu et al., 2001; Poedts et al., 2000; Popel et al., 2004a, b; Vladimirov and Ostrikov, 2004) often contain massive and heavily charged dust grains. The latter cannot only modify the characteristics of the plasma modes in corresponding dust-free plasmas but can also give rise to new modes. Because of charge redistribution, the presence of dusts in a plasma can also strongly affect its collective and transport properties. It is therefore of interest to investigate the behavior of low-frequency perturbations in inhomogeneous low-temperature dusty plasmas. In this paper we shall study the stability of a sheared rotational flow in a dusty plasma, in particular, the Rayleigh–Taylor (RT) instability (Chen, 1974; Sekar and Raghavarao, 1987; Tynan et al., 1992; Vranjes et al., 2003).

2. Basic equations

To keep the problem fairly general, we consider a low-β ($\beta \ll 1$) plasma containing electrons, ions, and negatively charged dust grains. The external magnetic field is given by $\mathbf{B}_0$ ($= B_0\mathbf{e}_z$) and the gravitational field by $\mathbf{g}$, which can also represent local curvature of magnetic field. The plasma is assumed to be inhomogeneous in the x direction. In the lower ionosphere, the electron temperature T_e is much

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higher than that of the ions, and for simplicity we set the ion temperature to zero (Verheest, 2000; Kopnin et al., 2004; Popel et al., 2004a, b). We shall concentrate on two-dimensional (2D) electrostatic perturbations perpendicular to $\mathbf{B}_0$. In the frequency regime ($\omega_{\rm cd} < \omega < \omega_{\rm ci}$, where $\omega_{\rm cd}$ and $\omega_{\rm ci}$ are the dust and ion cyclotron frequencies, respectively) of interest, the heavy dust grains can be taken to be immobile. We also assume that the plasma is sufficiently rarified such that the dust charge is constant, and that the quasi-neutral condition (Verheest, 2000; Popel et al., 2004a, b)

$$n_{\rm e}(x) + Z_{\rm d}n_{\rm d}(x) = n_{\rm i}(x) \tag{1}$$

is satisfied in the steady as well as the perturbed states. Here $n_{\rm e}$, $n_{\rm i}$ and $n_{\rm d}$ are the electron, ion, and dust densities, respectively, and $Z_{\rm d}$ is the constant dust charge.

The equations governing the behavior of the 2D RT mode are the continuity and perpendicular (to $\mathbf{B}_0$) momentum equations for electrons and ions together with the quasi-neutral condition (1). In the drift approximation ($d_t \ll \omega_{\rm ci}, \omega_{\rm ce}$), the electron and ion perturbation velocities perpendicular to the magnetic field are (Chen, 1974; Chakrabarti et al., 1994; Vranjes, 1999; Azeem and Mirza, 2006)

$$\mathbf{v}_{\perp \rm e} = \frac{c}{B_0}\mathbf{e}_z \times \nabla_\perp \varphi - \frac{cT_{\rm e}}{eB_0}\mathbf{e}_z \times \nabla_\perp \ln n_{\rm e}, \tag{2}$$

$$\mathbf{v}_{\perp \rm i} = \frac{c}{B_0}\mathbf{e}_z \times \nabla_\perp \varphi - \frac{c}{B_0\omega_{\rm ci}}d_t\nabla_\perp\varphi + \mathbf{V}_{\rm gi}, \tag{3}$$

respectively, where $d_t = \partial_t + (c/B_0)\mathbf{e}_z \times \nabla_\perp\varphi \cdot \nabla_\perp$, φ is the electrostatic potential, $\mathbf{V}_{\rm gi} = -(g/\omega_{\rm ci})\mathbf{e}_y$ is the gravitational, or magnetic-curvature, induced drift velocity. Substituting expressions (2) and (3) into the continuity equations for the electrons and ions, we obtain after some algebra the equations for the normalize electron and ion densities $\tilde{n}_{\rm e}$ and $\tilde{n}_{\rm i}$, and the normalized potential ϕ

$$d_t\tilde{n}_{\rm e} + \kappa\partial_y\phi = 0, \tag{4}$$

$$d_t(\tilde{n}_{\rm i} - \omega_{\rm ci}^{-1}\nabla_\perp^2\phi) + V_{\rm gi}\partial_y\tilde{n}_{\rm i} + \kappa\partial_y\phi = 0, \tag{5}$$

where now $d_t = \partial_t + \mathbf{e}_z \times \nabla_\perp\phi \cdot \nabla_\perp$. The normalized quantities are defined by $\phi = (c/B_0)\varphi$, $\tilde{n}_{\rm i} = n_{\rm i1}/n_{\rm i0}$, and $\tilde{n}_{\rm e} = n_{\rm e1}/n_{\rm e0} = \tilde{n}_{\rm i}/\varepsilon$. We have also defined $\varepsilon = n_{\rm e0}/n_{\rm i0}$, $\kappa = -\partial_x \ln n_{\rm i0}$, and used the quasi-neutrality condition.

Linearizing Eqs. (4) and (5), one obtains the dispersion relation for the RT mode

$$\omega^2 - (1-\varepsilon)\frac{\kappa k_y\omega_{\rm ci}}{k_\perp^2}\omega - \varepsilon\kappa\omega_{\rm ci}V_{\rm gi}\frac{k_y^2}{k_\perp^2} = 0 \tag{6}$$

in a dusty plasma. The corresponding instability condition for the 2D linear RT mode is then

$$g\kappa > \frac{\kappa^2\omega_{\rm ci}^2(1-\varepsilon)^2}{4\varepsilon k_\perp^2}, \tag{7}$$

which reduces to the well-known condition $g\kappa > 0$ for the RT instability in a simple two-component plasma. Thus, the addition of charged dust grains into a plasma can stabilize the RT instability. One can also easily show that the coupled nonlinear equations (4) and (5) admit quasi-stationary vortex solutions (Vranjes et al., 2003; Chen et al., 2003).

3. Derivation of the governing equation

Here, we are interested in flow-shear excited short-wavelength RT modes at the center of a vortex-like structure in a dusty plasma. Such an instability is referred to as a secondary instability by Chakrabarti (2000), since the background rotational motion or vortex may also have been caused by a similar, but of longer wavelength, RT instability.

In the presence of shear, a vortex-like rigidly rotating flow assumes an elliptic structural topology (Bayly, 1986; Sekar and Raghavarao, 1987). One can then approximate the 2D flow potential by (Bayly, 1986; Chakrabarti, 2000)

$$\phi_{\rm v}(x,y) = \frac{\Omega}{2}\left(\frac{x^2}{s} + sy^2\right), \tag{8}$$

where Ω is the rotation frequency and s is the eccentricity. The effective potential associated with the shear flow is $\phi_{\rm s} = V'_{\perp 0}x^2/2$, so that the total potential is $\phi(x,y) = \phi_{\rm v}(x,y) + \phi_{\rm s}(x,y) + \tilde{\phi}(x,y)$.

From Eqs. (4) and (5), we obtain for $\tilde{\phi}$ and $\tilde{n}_{\rm i}$

$$(\partial_t + V''_{\perp 0}x\partial_y - \Omega sy\partial_x)\tilde{n}_{\rm i} + \varepsilon\kappa\partial_y\tilde{\phi} = 0, \tag{9}$$

$$(\partial_t + V''_{\perp 0}x\partial_y - \Omega sy\partial_x)\left(\tilde{n}_{\rm i} - \frac{1}{\omega_{\rm ci}}\nabla_\perp^2\tilde{\phi}\right) + \kappa\partial_y\tilde{\phi} + V_{\rm gi}\partial_y\tilde{n}_{\rm i} = 0, \tag{10}$$

where $V''_{\perp 0} = V'_{\perp 0} + \Omega/s$. It should be noted that the potentials associated with flow rotation and shear are space dependent. For simplicity, hereafter the tilde sign in $\tilde{\phi}$ and $\tilde{n}_{\rm i}$ shall be dropped.

One can solve the inhomogeneous equations (9) and (10) using the Bayly's nonmodal method of variable separation (Bayly, 1986; Chakrabarti, 2000). Accordingly, we set

$$[\phi(\mathbf{r},t), n(\mathbf{r},t)] = [\phi(t), n(t)]\exp(\mathrm{i}\mathbf{k}_\perp(t)\cdot\mathbf{r}), \tag{11}$$

so that

$$d_tk_x(t) + V''_{\perp 0}k_y(t) = 0, \tag{12}$$

$$d_tk_y(t) - \Omega sk_x(t) = 0, \tag{13}$$

$$d_tn(t) + \mathrm{i}\varepsilon\kappa k_y(t)\phi(t) = 0, \tag{14}$$

$$d_t\left[n(t) + \frac{k^2(t)\phi(t)}{\omega_{\rm ci}}\right] + \mathrm{i}k_y(t)V_{\rm gi}n(t) + \mathrm{i}k_y(t)\kappa\phi(t) = 0, \tag{15}$$

where d_t now denotes the simple derivative.

From Eqs. (12) and (13), we obtain for the wave numbers

$$k_x(t) = k_0\sin(\overline{\Omega}t + \theta_0),$$