Available online at www.sciencedirect.com
ScienceDirect
Planetary and Space Science 55 (2007) 1388–1397
Planetary and Space Science
www.elsevier.com/locate/pss

Contraction of satellite orbits using KS uniformly regular canonical elements in an oblate diurnally varying atmosphere

M. Xavier James Raj*, R.K. Sharma

Vikram Sarabhai Space Centre, Applied Mathematics Division, Thiruvananthapuram 695 022, India

Received 3 October 2006; received in revised form 27 February 2007; accepted 2 March 2007
Available online 12 March 2007

Abstract

A new non-singular analytical theory for the motion of near Earth satellite orbits with the air drag effect is developed in terms of the Kustaanheimo and Stiefel (KS) uniformly regular canonical elements, by assuming the atmosphere to be oblate diurnally varying with constant density scale height. The series expansions include up to third-order terms in eccentricity and c (a small parameter dependent on the flattening of the atmosphere). Only two of the nine equations are solved analytically to compute the state vector and change in energy at the end of each revolution, due to symmetry in the equations of motion. Numerical comparisons of the important orbital parameters semimajor axis and eccentricity up to 1000 revolutions, obtained with the present solution, with the third-order analytical theories of Swinerd and Boulton and in terms of the KS elements, with respect to the numerically integrated values, show the superiority of the present solution over the other two theories over a wide range of eccentricity, perigee height and inclination.

Keywords: Contraction of satellite orbits; Non-singular analytical solution; KS uniformly regular canonical elements; Diurnally varying atmosphere

1. Introduction

Accurate orbit prediction of the near Earth's satellites is an important requirement for reentry and orbital lifetime estimates. The orbit decay of these satellites is mainly controlled by the atmospheric drag effects. The effects of the atmosphere are difficult to determine since the atmospheric density, and hence the drag undergoes large fluctuations.

The method of the Kustaanheimo and Stiefel (KS) total energy element equations (Stiefel and Scheifele, 1971) was employed to generate analytical solutions with atmospheric drag force with respect to exponentially oblate and oblate diurnally varying analytical atmospheric models (Sharma, 1991, 1992, 1997). Subsequently a particular canonical form of the KS differential equations, known as KS uniform regular canonical equations (KSC), where all the 10 elements are constant for unperturbed motion and the equations permit the uniform formulation of the basic laws of elliptic, parabolic and hyperbolic motion (Stiefel and Scheifele, 1971, p. 250) are used to generate analytical solution for short-term orbit predictions with Earth's zonal harmonic terms J_2, J_3, J_4 (Xavier James Raj and Sharma, 2003). Further, these equations were used to include the canonical forces and an analytical theory with air drag with spherically symmetrical exponential atmosphere was developed. The theory was compared with the KS theory (Sharma, 1992; Cook et al., 1961) over a wide range of perigee height, eccentricity and inclination and was found to be better than the other two solutions (Xavier James Raj and Sharma, 2006). An analytical theory with oblate exponential atmosphere was also developed with these uniformly regular KS canonical elements (Sharma and Xavier James Raj, 2005) and was found to be better than the KS theory (1992) and Swinerd and Boulton (1982) theory (when only atmospheric oblateness effects are introduced).

In the present paper, we have developed a non-singular analytical theory in terms of uniformly regular KS canonical elements with air drag using oblate diurnally varying atmosphere with constant density scale height. The series expansion method is utilized to generate the

*Corresponding author. Tel.: +91 471 2564636; fax: +91 471 2704134.
E-mail addresses: m_xavierjamesraj@vssc.gov.in (M. Xavier James Raj), rk_sharma@vssc.gov.in (R.K. Sharma).

0032-0633/$ - see front matter
doi:10.1016/j.pss.2007.03.002

Nomenclature

a	semimajor axis
A	effective area of the satellite
A_i, B_i	canonical drag forces
B	ballistic coefficient
c	parameter dependent on the flattening of the atmosphere
C_D	drag coefficient
$\mathbf{D}$	aerodynamic drag force per unit mass
e	eccentricity
E	eccentric anomaly
H_0	average density scale height
i	inclination
i_0	initial inclination
KS	Kustaanheimo and Stiefel
$L(u_i)$	KS matrix
L^T	transformation matrix of L
m	mass of the satellite
r	distance from the center of the Earth to satellite
r_{p_0}	initial perigee radius
s	new independent variable
t	time
$\mathbf{u}$	position vector in KS canonical variables
u_i	position components in KS canonical variables
$\mathbf{w}$	velocity vector in KS canonical variables
w_i	velocity components in KS canonical variables
$\mathbf{v}$	velocity vector
v	velocity
v_{p_0}	velocity at the initial perigee
x_i	position components
$\dot{x}_i$	velocity components
α_i, β_i	KS canonical elements
Λ	rotational rate of the atmosphere about the Earth's axis
Ω	right ascension of ascending node
ω	argument of perigee
ρ	atmospheric density
ρ_0	average density
ρ_{p_0}	density at initial perigee
ε	ellipticity of Earth
θ	geocentric latitude
ϕ	geocentric angular distance
μ	gravitational constant

analytical solutions and terms up to third order in eccentricity are retained. Only two of the nine equations are solved analytically to compute the state vector and change in energy at the end of each revolution, due to symmetry in the equations of motion and computation for the other equations (Stiefel and Scheifele, 1971, p. 91) is made by changing the initial conditions. For comparison purpose KS elements equations are integrated numerically with a fixed step size fourth-order Runge–Kutta–Gill method with a small step size of half degree in eccentric anomaly. Numerical experimentation over a wide range of perigee altitude, eccentricity and orbital inclination has been carried out up to 1000 revolutions. The numerical results obtained with the analytical solutions match quite well with the numerically integrated values and show improvement over the third-order theories of Swinerd and Boulton (1982) and Sharma (1997). The other analytical theories, in which the diurnal bulge and oblateness effect of the atmosphere are treated in combination, are by Santora (1975, 1976), Fominov (1976), Boulton (1983), Brookes (1992) and Nair and Sharma (2003).

2. Equations of motion

The KS canonical equations of motion with the canonical forces A_k, B_k are (Xavier James Raj and Sharma, 2006)

$$\frac{d\beta_k}{ds} = \frac{\partial H}{\partial \alpha_k} - B_k,$$
$$\frac{d\alpha_k}{ds} = -\frac{\partial H}{\partial \alpha_k} + A_k, \; (k = 0, 1, 2, 3, 4), \tag{1}$$

where the new independent variable 's' is defined by $dt/ds = r = u_1^2 + u_2^2 + u_3^2 + u_4^2$,

$$A_k = \sum_{j=0}^{4} W_j \frac{\partial u_k}{\partial \beta_j}, \; B_k = \sum_{j=0}^{4} W_j \frac{\partial u_k}{\partial \alpha_j}, \; (k = 0, 1, 2, 3, 4) \tag{2}$$

with

$$W_0 = -0.5\, r(\mathbf{v} \cdot \mathbf{D}),$$
$$W_j = -0.5\, r L^T(\mathbf{u}) \cdot \mathbf{D} \; (j = 1, 2, 3, 4), \tag{3}$$

$$x_i = L(\mathbf{u})\mathbf{u}, \quad \dot{x}_i = \frac{2}{r} L(\mathbf{u})\mathbf{w} \quad (i = 1, 2, 3), \tag{4}$$

$$L(\mathbf{u}) = \begin{pmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix},$$

$$u_0 = \beta_0 + \frac{1}{4}\sum_{k=1}^{4}\left[\begin{array}{l} \frac{1}{2\sqrt{\alpha_0^3}}(\alpha_0\alpha_k^2 - \beta_k^2)\sin(2\sqrt{\alpha_0}s) \\ + \frac{\alpha_k\beta_k}{\alpha_0}\{\cos(2\sqrt{\alpha_0}s) - 1\} + \frac{s}{\alpha_0}(\beta_k^2 + \alpha_0\alpha_k^2) \end{array}\right],$$

$$u_k = \frac{\beta_k}{\sqrt{\alpha_0}}\sin(\sqrt{\alpha_0}s) - \alpha_k\cos(\sqrt{\alpha_0}s),$$

$$w_0 = \alpha_0,$$
$$w_k = \beta_k\cos(\sqrt{\alpha_0}s) + \alpha_k\sqrt{\alpha_0}\sin(\sqrt{\alpha_0}s), (k = 1,\ 2,\ 3,\ 4), \tag{5}$$

$$\alpha_0 = \frac{1}{2}\left[\frac{\mu}{r} - \frac{v^2}{2}\right],$$

$$\alpha_k = \frac{w_k}{\sqrt{\alpha_0}}\sin(\sqrt{\alpha_0}s) - u_k\cos(\sqrt{\alpha_0}s),$$