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Analytical and graphical determination of the trajectory of a fireball using seismic data

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Abstract

In this paper we discuss two methods, one analytical and the other graphical, to determine the trajectory of a fireball using the arrival times of atmospheric shock waves recorded by a seismic network. In the analytical method the trajectory and the raypaths are assumed to be straight and we solve for the fireball velocity, the azimuth (φ) and elevation angle (δ) of the trajectory, the coordinates of the intersection of the trajectory with the earth's surface, and the corresponding intersection time (t_0). Because the problem is nonlinear, we solve it iteratively. The fireball velocity cannot be determined uniquely, and trades off with t_0. The graphical method is based on the drawing of contours of arrival times, which should be elliptical for fireball shock waves. If the distribution of seismic stations is appropriate, the horizontal projection of the fireball is given by the axis of symmetry of the contours, which allows the estimation of φ, while δ can be estimated from the spacing between contours along the symmetry axis. Application of the two methods to data from four fireballs shows that the graphically derived parameters can be within a few degrees of the analytical parameters. In addition, a fireball recorded in the Czech Republic has reliable trajectory parameters derived from video recordings, which allows an independent assessment of the quality of the parameters determined analytically. In particular, φ and δ have errors of 1.7° and 1.3°, respectively, which are not particularly large considering that the station distribution was not favorable.

Keywords: Fireball; Trajectory; Seismic; Shock waves

1. Introduction

Borovička et al. (2003) noted that of more than eight hundred documented meteorite falls, only six had a reliable determination of the velocity and trajectory of the fireball that preceded the fall. More recently, ReVelle et al. (2004) added another meteorite to the list. Given the importance of this information for studies of the solar system, increasing the number of well-determined fireball trajectories is highly desirable. Interestingly, some of this information is already being provided by analysis of the fireball shock waves recorded by seismic networks (e.g., Brown et al., 2003; Ishihara et al., 2003, 2004; Pujol et al., 2005). In recent years, the number and quality of seismic networks around the world has increased significantly and, as a consequence, the network data have the potential to become an important source of information to the astronomical community. In this paper we briefly describe two methods, one analytical and the other graphical, to determine the trajectory of a fireball using the arrival times of the atmospheric shock waves recorded at seismic stations (Pujol et al., 2005). The analytical method is iterative and has been designed to compute the velocity of the fireball, the azimuth and elevation angle of the trajectory (assumed straight), the coordinates of the intersection of the trajectory with the earth's surface, and the corresponding intersection time. The sound speed is assumed to be known and constant. Under appropriate conditions the azimuth, elevation angle, and the coordinates of the intersection are well constrained. The fireball velocity, however, cannot be determined uniquely; it

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depends on the initial value assigned to it and trades off with the intersection time. The graphical method requires the drawing of the isochrones, which may allow a quick computation of the azimuth and elevation angle of the trajectory. The two methods have been applied to data from fireballs recorded in the United States (Arkansas), Japan (Miyako and Kanto) and the Czech Republic (Morávka), and comparison of the results obtained using the two methods gives a good idea of the capabilities and limitations of the graphical method. In addition, the Morávka fireball has reliable trajectory parameters derived from video recordings, which allows an independent assessment of the quality of the parameters determined analytically.

2. Methods

2.1. Analytical method

Before applying this method it is necessary to derive the equation for the arrival time of the shock waves at points on the surface of the earth. This in turn requires a number of assumptions regarding the propagation of the waves, which usually are the following: (1) the fireball trajectory is a straight line, (2) its velocity (v) and the speed of sound (c) are both constant, and (3) there are no atmospheric winds. Because c is actually a function of the fireball height, an average value is used and the raypaths are assumed straight. As noted in the references given above, these assumptions are reasonable for the distances involved in the examples discussed here, which also allow neglecting the earth's curvature. As distance increases the assumption of straight rays is violated and the propagation model is no longer valid. It is possible, however, to improve on the approximations of constant c and absence of winds by computing average values of c depending on the fireball height (e.g., Brown et al., 2003) and by moving the stations by an amount opposite to the wind drift (Borovička and Kalenda, 2003). The effect of some of the approximations we used will be assessed in the context of the Morávka fireball, for which reliable parameters derived from optical data are available. As Fig. 1 shows, the trajectory is defined by the following parameters: the angles φ and ϑ and the coordinates $(x_0, y_0, 0)$ of its intersection with the earth's surface (point P). The elevation angle (δ) is equal to $90 - \vartheta$. Two other parameters to be determined are the velocity v and the time t_0 at point P. The wave generated at point T on the trajectory arrives at a station S with coordinates (x_s, y_s, z_s) at a time t given by

$$t = t_0 - \frac{d_t}{v} + \frac{d_p \cos\beta}{c}, \tag{1}$$

(Pujol et al., 2005) where d_t and d_p are distances along and perpendicular to the trajectory, respectively, equal to

$$d_t = \overline{PQ}, \quad d_p = \overline{SQ} \tag{2}$$

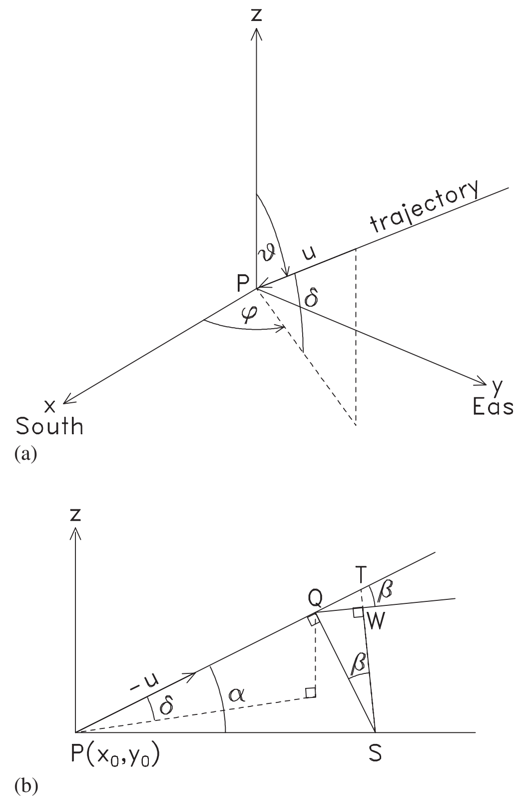

Fig. 1. (a) Local spherical coordinate system used to define the trajectory of a fireball. The system is centered at the intersection of the trajectory with the earth's surface (point P). The angle δ is the elevation of the trajectory and **u** is a unit vector. (b) Geometry for the derivation of Eq. (1). The dashed lines are as in (a). The wave that arrives at a point S on the earth's surface originates at the point T on the trajectory that makes the line $\overline{TS}$ perpendicular to the Mach cone, identified by its angle β. From Pujol et al. (2005).

(see Fig. 1) and β is the Mach angle, equal to

$$\beta = \sin^{-1}\frac{c}{v}. \tag{3}$$

Eq. (1) generalizes that used by Ishihara et al. (2003, 2004), which is valid for a rotated system with the z-axis along the trajectory and the x-axis in the vertical plane that contains the trajectory. For the computation of d_t and d_p the following relations are used:

$$\mathbf{u} = (\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, -\cos\vartheta) = (u_1, u_2, u_3), \tag{4}$$

$$\mathbf{b} = \overrightarrow{SP} = (x_s - x_0, y_s - y_0, z_s), \tag{5}$$

$$d_t = |\mathbf{b}|\cos\alpha = |\mathbf{b}\cdot\mathbf{u}|, \tag{6}$$

$$d_p = \sqrt{|\mathbf{b}|^2 - d_t^2}. \tag{7}$$

Before proceeding, it is worth noting that shock waves cannot be observed at all points on the ground. With reference to Fig. 1b, the waves cannot reach points to the left of a line passing through the point P and perpendicular to the projection of the trajectory. On the other hand, not