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A geometrization of electric charge and mass by means of a solution to the Einstein and Maxwell equations for dust and a radial electric field

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ABSTRACT

It has been shown that a physical system consisting of a free electromagnetic field and electrically neutral dust-like matter can be completely geometrized in 4-space-time. This system can be represented as worlds with a non-trivial topology, connected through an infinite set of the non-closing non-stationary wormholes. The geometric image of the electric charge turns out to be the non-closing static throat. The internal world of the charged particle is discretized into disconnected regions of space-time, impenetrable to light trajectories. The electric charge e, rest mass m_0, and classical action of the gravitational field have a gravitational nature and are expressed in terms of the metric and the space-time curvature.

1. Introduction

Geometrization of the physical fields is considered to be one of the ways of creating a unified field theory. Geometrization means the possibility to express the physical characteristics (mass, electric charge, fields strengths, their energy densities etc.) in terms of the geometric characteristics of space-time.

For a pseudo-Riemannian 4-space specified by its metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 0, 1, 2, 3, \quad signature\, g_{\mu\nu} = + - - -, \tag{1}$$

the expression of the physical characteristics in terms of the metric tensor $g_{\mu\nu}$ and its derivatives with respect to coordinates $\{x^\mu\}$ will be sufficient, since the connection $\Gamma^\mu_{\nu\lambda}$, the Riemann curvature tensor $R^\mu_{\bullet\nu\lambda\rho}$ as well as its contraction, i.e. the Ricci curvature tensor $R_{\mu\nu}$, and the divergenceless Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ [1, p.274] can be expressed in terms of these quantities.

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The same idea establishes the Einstein equation of general relativity in which the geometric tensor $G_{\mu\nu}$ is equated to the physical energy-momentum tensor $T_{\mu\nu}$ of these physical fields:

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = \frac{8\pi k}{c^4}, \tag{2}$$

where k is the gravitational constant. The energy-momentum tensor $T_{\mu\nu}$, in turn, contains the fields strengths, the energy densities, and the velocities of the mass and charge carriers.

Thus, all the physical fields curve space-time, and this curvature is perceived as the gravitational field, whose strength is identified with the Riemann curvature tensor $R_{\mu\nu\lambda\rho}$.

To complete the system of the differential equations, describing the mapping of physics onto geometry, the field equations, say, the Maxwell equations for the electromagnetic field:

$$F^{\mu\nu}{}_{;\nu} = -\frac{4\pi}{c} j^{\mu}, \tag{3}$$

have to be added to the Einstein equations. Here $F^{\mu\nu}$ is the electromagnetic field tensor, j^{μ} is the current density vector, $j^{\mu}=c\rho_f u^{\mu}$, $\rho_f=en_f$ is the electric charge density, e is the fundamental charge, n_f is the volume number density, and $u^{\mu}=\mathrm{d}x^{\mu}/\mathrm{d}s$ is the 4-velocity of the charged matter.

Since there is a geometric identity $G^{\mu\nu}{}_{;\nu}=0$, the system of Eqs. (2), (3) contains also the equations of motion of the mass and electric charge carriers [1, p.256]:

$$\varepsilon_s \frac{Du^{\mu}}{ds} = \rho_f F^{\mu\nu} u_{\nu}, \tag{4}$$

where $\varepsilon_s=m_0c^2n_s$ is the matter energy density, m_0 is the rest mass of the elementary mass carriers, and n_s is their volume number density. For homogeneous matter $n_f=n_s$.

Thus, it seems to be sufficient to express all the physical characteristics of the electromagnetic field: ε_f, ε_s, ρ_f, $F^{\mu\nu}$, u^{μ}, m_0, e, in terms of the geometric ones: $g_{\mu\nu}$, G^{ν}_{μ} and the others from the Eqs. (2)–(4) for a geometrization of electromagnetic field[1].

In the main part of this paper, we will show how it can be done by the simplest example of the spherically symmetric system consisting of electrically neutral dust-like matter and a free electromagnetic field, which is merely the radial electric field in the comoving to matter reference frame.

However such a geometrization cannot be considered complete, since we have not described geometrically what the geometric source of the electromagnetic field and the common matter is, namely, the fundamental charge e and the rest mass m_0 of the charged particle, like the electron and proton[2] (we will consider the neutral mass carriers to consist of the charged ones).

In the existing theories of electromagnetism, the electric charge carriers are crucially point entities in both classical and quantum electrodynamics. The formal consequence of this fact is a singularity, namely, the Coulomb divergence of the point charge field, which is present in all such models.

In contrast to this point of view, we suppose that the elementary carriers of the electric charge and the rest mass are non-pointlike, i.e. they have the characteristic size of the order of the classical radius

$$r_f = \frac{e^2}{2m_0c^2}, \tag{5}$$

in observing from the vacuum, where e and m_0 are the charge and rest mass of the electron, or proton, or any more complex charged particle.

The most important feature of our suggestion is that the elementary carriers of the electric charge are holes in the observed space-time, whose inner world does not belong to our space but belongs to another space connected with ours by this narrow throat, which is what we consider as a point charge carrier (in special relativity).

How can this inner world be found? If the gravitational interaction is considered to be universal, that is, firstly, “covering”, “resulting” in all the physical interactions, mapping them on the space-time curvature. Secondly, it is manifesting itself at any scale (in the micro-, macro-, and mega-world)[3], then we can try to find the internal compact exact solution to the equations of the considered gravitational electrodynamic model, containing such a narrow throat without singularities, that will be “glued” in the observed world as the electric charge.

Such a solution to the Einstein and Maxwell equations has been obtained in [2]. It will be described in one of the following sections of this paper. It has been obtained for the simplest non-stationary spherically symmetric system consisting

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[1] Methodically, it is evidently not correctly to say about the equality of the physical fields (electromagnetic one, nuclear one, and others) and the gravitational field, since the electromagnetic field is a definite state of the space-time curvature, that is, the gravitational field. In other words, the gravitational field has a “covering” nature.

[2] considering elementary particles as classical objects.

[3] In the classical model in general relativity (before the solution of its equations), the interaction length is not bounded from below and above. This is true both for the quantum Planck length $r_{pl} = \sqrt{\frac{\hbar c^3}{k}} \sim 10^{-33}$ cm and the critical length $r_c = \frac{e\sqrt{k}}{c^2}$, which is less than the first one by a factor of $1/\sqrt{\alpha}$, where $\alpha = \frac{e^2}{\hbar c}$ is the fine-structure constant.