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Investigation of light baryons in a simple analytical treatment

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ABSTRACT

We present a simple analytical solution of three-quark problem for investigating the mass spectrum of light baryons. We use the well-known "Coulombic-plus-linear" potential plus spin and isospin exchange interactions to reproduce the hyperfine structure of the baryon. Performing calculations in hypercentral approach without any perturbative approximation, we obtain the spin- and isospin-dependent energy levels and eigenfunctions for a three-quark system. Nevertheless, the description of the baryon spectrum in our model is comparable with the experimental spectrum.

1. Introduction

The internal structure of baryons is well described in Constituent Quark Models, where the baryons are considered as a system of three constituent confined quarks [1–3]. As a useful approach for an efficient study of baryon properties, the hypercentral Constituent Quark Model consists of a hypercentral quark interaction containing a linear plus coulomb-like term, as suggested by lattice QCD calculations [4–6]. Inspired by Lattice QCD calculations [7,8] and the experimental baryon spectrum (which shows an underlying SU(6) symmetry in the spectrum of the observed baryon resonances), the three-quark interaction potential in Constituent Quark Models contains two main terms: a dominant confining SU(6) invariant term which accounts of the spacing between each SU(6) multiplet and a small SU(6) breaking term which have to describe the splittings within each multiplet [6,9]. Total wave functions are constructed by means of the tensor products of spatial, spin, isospin (flavor) and color wave functions, *i.e.* $\Psi_{3q} = \Psi_{\text{space}}\ \chi_{\text{spin}}\ \Theta_{\text{isospin}}\ \theta_{\text{color}}$. According to the standard three-quark Constituent Quark Model, the total antisymmetry (with respect to quark interchange) is given by the color factor Φ_{color} and the product $\Psi_{\text{space}}\ \Phi_{SU(6)}$ must be symmetric. So, a non-relativistic constituent quark model has been proposed in the hypercentral approach in [10] for main SU(6)-invariant part. Inspired by the apparent success of the hypercentral Constituent Quark Model, we assumed a spin- and isospin-independent SU(6)-invariant interaction, and spin- and isospin dependent interactions as the SU(6)-breaking interaction, to investigate the baryons as a three-quark system within a simple analytic solution. There exist a significant different between the present work and hypercentral Constituent Quark Model [10] and it is that we do not consider the SU(6)-breaking interaction as a perturbation. However, the description of the nonstrange baryon spectrum obtained by our model based on a hypercentral Constituent Quark Model is generally fair. This paper is

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organized as follows. In Section 2, we present our model (based on hypercentral constituent quark model) through which we obtained analytical formulas for spin and isospin dependent energy levels and the hyperradial wave functions. In Section 3, we present our results for the detailed structure of the spectrum and finally a conclusion has been provided.

2. The model

In this section, we aim to investigate the mass spectrum of light baryons within a simplified analytical solution of three-quark problem without applying any perturbative approximation. In present work, we consider the baryons as bound states of three constituent (massive) quarks, in the baryon rest frame and, for simplicity, the three constituents have been considered identical, $m_u = m_d$. After having removed the center of mass coordinate, $\mathbf{R} = 0$, the internal quark motion is described by the Jacobi coordinates that are defined as:

$$\boldsymbol{\rho} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}, \quad \boldsymbol{\lambda} = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}} \tag{1}$$

The ρ and λ are variables describing respectively the relative position of the pair in the two-body subsystem, and the position of the third particle relative to the center of mass of the pair. The hyperspherical coordinates for describing the three-body problem are given by the angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$ together with the hyperradius x and the hyperangle ξ which are defined as follows [11–13]:

$$x = \sqrt{\rho^2 + \lambda^2}, \; \xi = \arctan\left(\frac{\rho}{\lambda}\right) \tag{2}$$

The hyperradius x is a collective space coordinate which is symmetric under any permutation of the three particles. On the center of mass framework in which total momentum is zero, we can write the kinetic energy operator of a three-body problem by using the hyperspherical coordinates, in six-dimensions as [14,15]:

$$T = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} + \frac{L^2(\Omega_5)}{x^2}\right) \tag{3}$$

The term $L^2(\Omega_5)/x^2$ which is a generalization of the centrifugal barrier for the case of the six-dimensional space involves the angular coordinates Ω_ρ, Ω_λ and hyperangle ξ. For the three-particle system, the eigenvalues of $L^2(\Omega)$ are given by $L^2(\Omega_5)Y_{[\gamma]}(\Omega_5) = -\gamma(\gamma+4)Y_{[\gamma]}(\Omega_5)$. Here γ, the degree of the polynomial, is called the grand orbital quantum number and is equal to $2n + l_\rho + l_\lambda$, where $l_\rho(l_\lambda)$ is the angular momentum corresponding to the coordinates $\rho(\lambda)$ and $n = 0, 1, 2, \ldots$ [16]. In the hypercentral constituent quark model (hCQM), the quark potential is assumed to be hypercentral and hence depends only on the hyperradius x. Because of $x \equiv x(\rho, \lambda)$, the hypercentral potential $V(x)$ is not an entirely a two-body interaction but it contains three-body contributions. Since the potential depends on x only, in the three-quark wave function we can factor out the hyperangular part, which is given by the known hyperspherical harmonics [13]. The remaining hyperradial part of the wave function is determined by the hypercentral Schrödinger equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_{\nu\gamma}(x) = -2m[E - V(x)]\psi_{\nu\gamma}(x) \tag{4}$$

where m is the common quark mass and $\psi_{\nu\gamma}$ is the space wave function of the three-quark system in which ν and γ are the quantum numbers describing the hyperradial excitations and grand orbital angular momentum (of three quark state), respectively. For this sector, we consider the interaction potential of quarks inside the baryon composed by a linear confining term, dominant at large separations, and a one-gluon-exchange (OGE) term, dominant at short separations, given by a central Coulomb-like potential and a spin- and isospin-dependent part, responsible for the hyperfine splitting of baryon masses:

$$V_{3q} = \sum_{i<j} ar_{ij} - \frac{b}{r_{ij}} + \frac{e^{-\mu r_{ij}}}{r_{ij}}\left[c\left(\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\right) + d\left(\boldsymbol{\tau}_i.\boldsymbol{\tau}_j\right) + e\left(\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\right)\left(\boldsymbol{\tau}_i.\boldsymbol{\tau}_j\right)\right] \tag{5}$$

Here, we smeared out the δ-function in spin and isospin exchange interactions by a Yukawa-type factor. By considering wave function as $\psi_{\nu\gamma}(x) = x^{-5/2}\,\phi_{\nu\gamma}(x)$, Eq. (4) with hypercentral potential (5) results in:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{(2\gamma+3)(2\gamma+5)}{4x^2} + 2mE - 2m\left(\alpha x - \frac{\tau}{x} + \frac{e^{-\mu x}}{x}A_{\text{hyp}}(S,T)\right)\right)\phi_{\nu\gamma}(x) = 0 \tag{6}$$

In which

$$A_{\text{hyp}}(S,T) = A_S\left(S^2 - \frac{9}{4}\right) + A_I\left(T^2 - \frac{9}{4}\right) + A_{\text{SI}}\left(S^2 - \frac{9}{4}\right)\left(T^2 - \frac{9}{4}\right) \tag{7}$$

Since, the Eq. (6) cannot be solved exactly, we shall use an approximation in order to deal with the problematic terms of hypercentral potential. In an approximative approach, we used approximations in order to deal with the linear confining and Coulombic-like terms. For this end, first we change the variable $t = \frac{1}{x}$ and then we employed the following approximation for the $\frac{\alpha}{t}$. We suppose that there is a characteristic radius of the baryon, x_0. The approximation is based on the expansion