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Stationary statistical properties of nonlinear energy harvesting system under high-pass filter

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ABSTRACT

In the present paper, we investigate stationary statistical properties of a nonlinear energy harvesting system under the high-pass filter and give the stationary probability distribution function $P_{st}(x, y)$ of the nonlinear energy harvesting system. We discuss effects of the system parameters on the probability distribution function $P_{st}(x, y)$ and the mean of the square of the output electrical voltage $\langle V^2 \rangle$ of the nonlinear energy harvesting system under the high-pass filter. When the exponent of the potential function variable is odd numbers, the probability distribution function $P_{st}(x, y)$ is insignificant. When the exponent of the potential function variable is even numbers, the probability distribution $P_{st}(x, y)$ shows two-maximum structure, and the peak value of the probability distribution decreases as the even exponent increases. The noise strength attenuates the peak value of the probability distribution $P_{st}(x, y)$, which reduces the stability of the system output. The noise strength makes the mean of the square of the output electrical voltage $\langle V^2 \rangle$ exhibit one-minimum structure, which is ant-coherence resonance induced by noise. The mean of the square the output voltage $\langle V^2 \rangle$ decreases as the even exponent increases.

1. Introduction

With the development of human society and the progress of science and technology, the use of the remotely distributed wireless microsensor is more and more popular and extensive. The new and efficient powering of small portable electronic devices should urgently be solved. It is desirable highly for us to have power sources co-located with microdevices here and when necessary, namely, one wish extremely to achieve efficiently the conversion from the almost ubiquitous ambient energy to the electronic energy by the piezoelectric, electromagnetic, or capacitive methods, whether this be electro-magnetic (light), thermal, mechanical, or other energy in nature, which is the so-called ambient energy harvesting. The research on the project is a very challenging task possessing wide applied prospects [1–3]. The traditional approach is based on the resonant tuning of the linear mechanical oscillator [4–8]. However, the geometrical factors of the oscillator and the prevalence of the energy spectra distribution of low frequency components restrain the efficiency of the ambient energy harvesting. If we use nonlinear oscillator instead of linear, i.e., resonant, ones, we can break through the limits of these conditions, so that we can efficiently enhance the efficiency of the ambient energy harvesting [9–13]. Recently, some researchers make the experiment of a piezoelectric oscillator by considering non-linear oscillators instead of the linear ones and construct

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the effective stochastic dynamics model consisting of the potential function, the output voltage, the dynamical variable, the oscillator velocity, the stochastic forces, and the relevant parameters, which is also the so-called "nonlinear energy harvesting" [14–25]. Discussing the statistical properties of the nonlinear energy harvesting system, we can further understand the effects of internal and external factors on the stochastic system so that find the approach optimizing and enhancing the output power of the ambient energy harvesting system.

In this paper we investigate stationary statistical properties of nonlinear energy harvesting system under the high-pass filter, give the stationary probability distribution function of the dynamical variable and the oscillator velocity, and discuss the effects of the relevant parameters of the system on the stationary probability distribution and the mean of the square of the output electrical voltage. The paper is organized as follows: In Section 2, we discuss the stochastic coupled equations of the nonlinear energy harvesting system under the high-pass filter and give the stationary probability distribution function $P_{st}(x, y)$. In Section 3, we discuss effects of the noise strength and relevant parameters of the system on stationary probability distribution $P_{st}(x, y)$ and the mean of the square of the output electrical voltage $\langle V^2 \rangle$. Finally, in Section 4, summaries and conclusions of the results conclude the paper.

2. The stochastic coupled equations and stationary probability distribution function

The dynamic coupled equations of a piezoelectric oscillator of the inverted pendulum system for nonlinear energy harvesting can be denoted by [13–15]

$$\frac{d^2x}{dt^2} = \frac{dU}{dx} - \gamma \frac{dx}{dt} - K_v V + \xi(t), \tag{1}$$

$$\frac{dV}{dt} = K_c \frac{dx}{dt} - \frac{1}{\tau_p} V, \tag{2}$$

where $U(x)$ is the potential energy function of the inverted pendulum system, x represents the relevant oscillator dynamical variable, V is the value of the voltage drop converted by nonlinear energy harvesting, γ is the viscous friction coefficient, K_v is the coupling coefficient that relates the oscillation to the voltage, $\xi(t)$ is the ambient force mimicked by the random force, K_c is the coupling constant of the piezoelectric sample, and τ_p is the time constant of the piezoelectric dynamics which related to the coupling capacitance C and to the resistive load R by $\tau_p = RC$.

Performing the transform $dx/dt = y$, one can obtain the Langevin equations corresponding Eqs. (1 and 2) that can be rewritten as

$$\frac{dx}{dt} = y, \tag{3}$$

$$\frac{dy}{dt} = \frac{dU}{dx} - \gamma y - K_v V + \xi(t), \tag{4}$$

$$\frac{dV}{dt} = K_c \frac{dx}{dt} - \frac{1}{\tau_p} V. \tag{5}$$

For the sake of simplicity and without loss of generality, we take the random force $\xi(t)$ to be zero-mean Gaussian white noise, whose statistical properties are

$$\langle \xi(t) \rangle = 0, \tag{6}$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t'), \tag{7}$$

where D is the noise strength.

The Fokker–Planck equation (FPE) for the probability distribution $P(x, y, V, t)$ of the dynamical variable x and the oscillator velocity y corresponding the three-dimensional Langevin Eqs. (3)–(5) is given by

$$\begin{aligned}\frac{\partial P(x, y, V, t)}{\partial t} = &-\frac{\partial [F(x, y, V)P(x, y, V, t)]}{\partial x} - \frac{\partial [B(x, y, V)P(x, y, V, t)]}{\partial y} \\ &- \frac{\partial [C(x, y, V)P(x, y, V, t)]}{\partial V} + D\frac{\partial^2}{\partial y^2}P(x, y, V, t),\end{aligned} \tag{8}$$

where

$$F(x, y, V) = y, \tag{9}$$

$$B(x, y, V) = \frac{dU}{dx} - \gamma y - K_v V. \tag{10}$$